Growth with Endogenous Industry Dynamics

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Abstract

This paper develops a Schumpeterian growth model with endogenous industry dynamics. A continuum of industries is endogenously created, evolves through a life cycle, and eventually exits. Potential entrants choose between joining an existing industry to compete for market share or paying a sunk cost to create a new industry and secure a temporary monopoly. This entry margin governs the balance between intensifying within-industry competition and expanding technological variety, thereby endogenously determining industry turnover and the composition of growth. In Markov-perfect equilibrium, individual industries follow non-stationary life-cycle paths, yet the economy exhibits stationary aggregate growth with continual sectoral rotation. I identify a new set of externalities operating through the industry-creation margin that distort equilibrium entry and innovation incentives. Using quantitative methods, I characterize comparative statics with respect to startup formation and industry-creation costs, and calibrate the model to U.S. data from 1982–2011. The calibrated economy exhibits insufficient industry renewal in equilibrium and, consequently, subsidies targeted at variety creation deliver larger gains in growth and welfare than traditional subsidies to incumbent R&D.

Keywords: Schumpeterian growth; Endogenous industry dynamics; Industry life-cycle; Variety creation; Innovation policy

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1 Introduction

In his presidential address at the 1998 annual meeting of the American Economic Association, Harberger (1998) contrasted two views of the growth process: a "yeast" view, in which productivity rises broadly and proportionately across sectors, and a "mushroom" view, in which growth comes from a rotating set of high-performance industries that sprout, flourish, and fade. The empirical record strongly supports the mushroom view of growth. Productivity gains are concentrated in a small set of sectors whose identity changes over time, rather than being spread uniformly across the economy (Ferguson and Wascher, 2004; Sarte and Taylor, 2025). At any point in time, only a few industries account for a large share of aggregate TFP growth, and the leaders are not persistent.

A large empirical literature on industry and product life-cycles provides a natural micro-level counterpart to this perspective. Young industries typically exhibit high entry, rapid product innovation, and volatile market shares. As they mature, entry slows, concentration rises, and innovation becomes more incremental (Gort and Klepper, 1982; Abernathy and Utterback, 1978; Jovanovic and MacDonald, 1994; Klepper, 1996; Filson, 2001; Argente, Lee and Moreira, 2024). In other words, industries tend to surge early and then slow down, so the industries that lead in TFP contribution can change as cohorts mature and new ones emerge.

Yet aggregate productivity growth in advanced economies has remained relatively steady over long horizons. This tension between micro-level turbulence and macro-level stability is at the core of the questions this paper addresses. How do non-stationary life-cycles of individual industries aggregate into a balanced growth path for the economy as a whole? What determines the rate at which new "mushrooms" appear to replace mature, low-growth sectors? How do entry and innovation policies affect the balance between improving existing products and creating entirely new markets?

These questions matter for policy because the same intervention can operate very differently across life-cycle stages. Industries at different stages face different competitive environments, innovation incentives, and entry margins. A policy that stimulates R&D in young, rapidly expanding industries may have very different effects in mature, follower-heavy industries where business stealing is severe and innovation incentives are weak. Understanding how industry evolution and entry decisions interact with innovation policy is therefore crucial for assessing the aggregate impact of R&D subsidies and policies that affect market entry and variety creation.

Many standard endogenous growth models sidestep this issue by fixing the set of product lines or treating industry creation as exogenous. This paper takes a different approach. I develop and quantify a step-by-step Schumpeterian growth model in which the set of active industries is endogenous. The economy features a continuum of industries that are created, evolve, and exit over time. Within each industry, firms conduct quality-improving R&D in a step-by-step fashion and compete in prices. A key decision margin is the one faced by the entrants. Upon arrival, an entrepreneur can either join an existing industry and compete for incumbent market share, or pay a sunk cost to create a new industry and obtain a temporary monopoly position. This choice determines the balance between intensifying competition within incumbent markets and expanding the variety of industries in the economy. Through this channel, entry shapes the cross-sectional distribution of industries over life-cycle states and the long-run growth rate.

The model generates industry life-cycles endogenously. New industries begin as small monopolies with a one-step lead over the competitive fringe. As they prove profitable, they attract followers, transition into oligopoly, and experience high innovation rates. Over time, leaders pull ahead, followers catch up less frequently, and the incentives for further entry and follower innovation decline. Mature industries become larger but less dynamic, and joining them becomes less attractive for new entrepreneurs. At that point, potential entrants are more likely to bear the cost of creating new industries instead. In equilibrium, each industry follows a non-stationary path, but the cross-sectional distribution of industries over states is stationary, so aggregate growth is balanced. The economy continuously renews itself through creation and evolution of industries, in line with Harberger's mushroom view.

This paper makes two main contributions. First, I develop a step-by-step Schumpeterian growth model in which industries follow endogenous life-cycles and aggregate growth emerges from the overlapping dynamics of these industries. Industries begin as monopolies, transition to oligopoly with intense competition and leadership turnover, and eventually mature as technology gaps widen and innovation incentives fall. Despite this non-stationary behavior at the industry level, the economy admits a stationary Markov-perfect equilibrium with an invariant cross-sectional distribution of industries and a constant long-run growth rate. This structure yields an explicit decomposition of growth into contributions from variety creation, frontier innovation by leaders, and catch-up innovation by followers, weighted by the stationary distribution over life-cycle states.

Second, the model implies new policy-relevant wedges because entry affects not only within-industry competition but also the economy's renewal rate. Entrants do not internalize how their choice between entering an incumbent industry and creating a new one reshapes the stationary distribution of industries across life-cycle states, which feeds back into aggregate growth and innovation incentives. Entry is further distorted by congestion and the resource cost of market creation, by the knowledge gains from seeding new industries that are not fully appropriable, and by the creative-destruction hazard that raises private discounting relative to the planner. I discipline the magnitudes by calibrating the model to U.S. data from 1982–2011 and use the calibrated economy to trace how entry and competition shift growth across margins. In particular, startup entry has a U-shaped relationship with growth, and tougher product-market competition raises growth primarily by reallocating entrants away from mature industries and toward creating new ones. These channels shape the policy ranking: in equal-cost experiments, subsidies targeted at variety creation generate larger gains in long-run growth and welfare than subsidies to incumbent R&D because they operate directly on the renewal margin.

Related Literature This paper contributes to the endogenous growth literature. I build on the Schumpeterian growth framework in which innovation is step-by-step and driven by patent races (Aghion et al., 2001; Grossman and Helpman, 1991; Akcigit and Ates, 2021, 2023). Relative to canonical step-by-step frameworks, I allow follower technology to improve gradually rather than through instantaneous leapfrogging (Liu, Mian and Sufi, 2022). I also do not add an exogenous catch-up term to followers' innovation rates. Instead, stationary aggregate growth is sustained by the extensive margin of endogenous variety creation, which makes new industries' early life-cycle innovation compensate for the slowdown of mature cohorts.

Among recent contributions, Olmstead-Rumsey (2019) and Cavenaile, Celik and Tian (2023) are closest in terms of modeling within-industry competition and innovation in Schumpeterian environments. Olmstead-Rumsey (2019) study how changes in the distribution of idea quality affect innovation outcomes and market structure, and use the framework to interpret recent trends in concentration. Cavenaile, Celik and Tian (2023) develop an oligopolistic Schumpeterian model in which the number of dominant firms within an industry is endogenously determined and firms choose innovation intensities strategically, with implications for concentration and business dynamism. My contribution is to add an endogenous extensive margin of industry creation that determines the stationary cross-sectional mix of young and mature industries. This renewal margin is central for policy, because private entrants do not internalize how their entry location reshapes the long-run distribution of industries and, through it, aggregate growth and welfare.

The paper also relates to the literature on industry and product life-cycles (Agarwal, 1998; Jovanovic and MacDonald, 1994; Klepper, 1996; Filson, 2001; Jovanovic and Wang, 2024). The literature has confirmed several stylized life-cycle patterns including the increasing industry-level output with decreasing prices (Gort and Klepper, 1982), decreasing product-level output (Argente, Lee and Moreira, 2024), decreasing firm entry rate (Agarwal and Gort, 1996), and decreasing innovation frequencies (Abernathy and Utterback, 1978), and develops partial-equilibrium models to account for them. I contribute by embedding life-cycle dynamics into a general equilibrium growth framework with endogenous industry creation and a stationary cross-sectional distribution of industries over life-cycle states.

Finally, this paper relates to work on innovation policy and externalities in endogenous-growth environments (Jones and Williams, 2000; Bloom, Schankerman and Van Reenen, 2013; Atkeson and Burstein, 2019; Acemoglu et al., 2018; Akcigit, Hanley and Stantcheva, 2022). Closest in spirit, Acemoglu and Cao (2015) develop a Schumpeterian model with incremental innovation of incumbents and radical innovation of entrants, and show that subsidizing entrant innovation can have limited growth effects when entry depresses incumbent innovation incentives. My contribution is to identify and quantify externalities that arise specifically from endogenous industry creation and to show how they reshape the welfare comparison between subsidies to incumbent R&D and subsidies to entry and variety creation. In the calibrated model, potential entrants undervalue variety creation relative to business stealing, so the decentralized equilibrium is biased against renewal. As a result, in equal-cost experiments, subsidies targeted at variety creation deliver larger gains in long-run growth and welfare than subsidies targeted at incumbent R&D because they operate directly on the renewal margin.

The rest of the paper is organized as follows. Section 2 reviews stylized facts on industry life-cycles and sectoral contributions to aggregate growth. Section 3 lays out the model. Section 4 characterizes the stationary Markov-perfect equilibrium, derives the decomposition of aggregate growth, and analyzes the constrained planner's problem on the industry creation margin. Section 5 presents the quantitative analysis and policy experiments. Section 6 concludes.

2 Motivating Evidence

This section summarizes four empirical patterns that motivate the framework and flesh out the Harberger's "mushroom" view of growth discussed in the introduction. Each pattern highlights a different aspect of the tension between non-stationary industry dynamics and relatively stable aggregate growth, and points to the importance of modeling industry creation and life-cycles explicitly.

The first pattern is that, over long horizons, aggregate productivity growth is relatively stable even as the industries that account for that growth change substantially over time. Ferguson and Wascher (2004) show that the U.S. has experienced several distinct productivity booms, each associated with different leading sectors. Using postwar U.S. data, Sarte and

Taylor (2025), Jorgenson, Ho and Stiroh (2008), and Oliner, Sichel and Stiroh (2007) show that the industries making the largest contributions to aggregate productivity growth have shifted over time—from durable goods manufacturing to IT-producing industries, and then toward IT-using service industries later. More recently, Hobijn et al. (2025) use quarterly industry-level data since 2008 to show that the main contributors to U.S. labor productivity growth have continued to rotate across industries before and after the pandemic, while average labor productivity in the nonfarm private sector has remained close to a stable pre-Covid linear trend. In the model, the rotating set of high-growth industries arises from a stationary cross-sectional distribution of industries over life-cycle states. Industries move through non-stationary paths, but the population shares in each state remain constant, delivering balanced aggregate growth.

The second pattern is that, at the aggregate industry level, output tends to rise while prices decline, whereas individual product- or brand-level quantities decrease. Gort and Klepper (1982) and Agarwal (1998) document that, across a wide range of product markets, diffusion and industry evolution are typically associated with increasing quantities and falling prices, and that this combination of rising physical output and falling prices is a robust feature of the industry life-cycle in many markets. At the product level, by contrast, quantities tend to quickly peak and then decline, with products frequently replaced by new varieties (Argente, Lee and Moreira, 2024). Thus, growing industries with declining prices coexist with shrinking or disappearing products—a contrast that the model captures through overlapping product and industry life-cycles. The model captures this contrast through overlapping life cycles where industries expand through entry and innovation even as individual varieties are displaced over time.

The third pattern concerns firm demographics over the industry life-cycle. In many industries, the number of active firms rises strongly in the early stages and later levels off or declines, while entry intensity falls as the industry matures (Gort and Klepper, 1982; Agarwal, 1998). Using a larger set of product markets with explicit entry and exit data, Agarwal and Gort (1996) show that industry entry rates peak in the early growth stages and then fall over time, while exit rates rise later in the life-cycle, so that the number of firms grows rapidly at first and then stabilizes or, in many markets, eventually turns down. For the purposes of this paper, the key feature is the declining entry rates in more mature industries, rather than the eventual shakeout. The model captures this through an endogenous entry decision. As industries become more mature and profitable opportunities shrink, expected entry returns fall and fewer new firms choose to enter.[1]

[1] To keep the analysis tractable, I do not model the late-stage decline in the number of firms. A standard extension with fixed operating costs could generate such a shakeout without changing the main mechanisms.

The fourth pattern is that innovation activity is front-loaded in the industry life-cycle: product innovation is especially intense early on and declines as industries mature. Drawing on multiple case studies, Abernathy and Utterback (1978) argue that new industries begin in a "fluid" phase with frequent major product innovations, after which a dominant design emerges and innovation shifts toward incremental process improvements. Abernathy and Utterback (1978) and Klepper (1996) document similar trajectories across a range of industries, with high rates of new product introduction and technological experimentation in the early stages, followed by lower rates of product innovation and a greater focus on cost-reducing process innovation once the market structure stabilizes. Quantitative evidence in Agarwal (1998) shows that patenting activity tends to rise in the early years of a product market and then decline in later stages, indicating an eventual slowdown in technological activity. In the model, this shows up as a declining innovation rate at the industry level, even as existing varieties continue to be produced and new industries continue to appear.

Taken together, these patterns point to a growth process in which aggregate regularities emerge from the interaction of many industries at different life-cycle stages, rather than from a representative sector, and motivate the model developed in the next section.

3 Model

The model builds on a step-by-step Schumpeterian growth framework (Akcigit and Ates, 2021, 2023; Liu, Mian and Sufi, 2022; Olmstead-Rumsey, 2019; Cavenaile, Celik and Tian, 2023) with heterogeneous industries and firm-level innovation. Time is continuous and indexed by $t \in [0, \infty)$. The economy is populated by a representative household that consumes, saves, and supplies labor, and a continuum of industries indexed by $i \in [0, N_t]$. The mass of active industries N_t is endogenous and changes over time as entrepreneurs create new markets and existing industries exit.

The central feature of the model is the *endogenous industry life-cycle*. Industries are created by entrepreneurs, evolve through phases of monopoly and oligopolistic competition driven by step-by-step innovation, and eventually exit due to exogenous obsolescence. Unlike standard Schumpeterian models with a fixed set of product lines, *potential entrants* in this economy face a key strategic choice between two margins. They can enter an incumbent industry to compete for existing market share, or they can create a new industry and obtain a temporary monopoly position. This choice between intensifying competition and expanding variety governs both aggregate growth and the structural evolution of the economy.

I first describe households and the final-good technology, then the structure of industries, and finally the innovation, entry, and exit processes. Throughout the section I state the assumptions explicitly and keep notation close to the economic objects they represent.

3.1 Household

A representative household has logarithmic preferences over a final consumption good. The household maximizes lifetime utility

$$U_t \;=\; \int_t^{\infty} e^{-\rho(s-t)} \log C_s \, ds, \tag{1}$$

where $\rho > 0$ is the rate of time preference and C_t denotes consumption at time t. The household owns the aggregate asset A_t and supplies one unit of labor inelastically to the labor market. Its budget constraint is

$$C_t + \dot{A}_t \;=\; w_t + r_t A_t, \tag{2}$$

where w_t is the wage and r_t is the interest rate.

The final consumption good, which serves as the numeraire, is produced with a Cobb-Douglas aggregator over industry outputs:

$$Y_t \;=\; N_t \exp\left(\frac{1}{N_t} \int_0^{N_t} \ln Y_{it} \, di \right),$$

where N_t is the mass of industries at time t and Y_{it} is the output of industry i. The aggregator implies that each industry i receives the same revenue flow $\bar{Y}_t \equiv Y_t / N_t$.

3.2 Industries and Production

Unless noted otherwise, I omit time subscripts for brevity. Each industry consists of a finite number of firms. If an industry has only one firm, it is a *monopoly*. If there are multiple firms, the industry is an *oligopoly* with one *leader* (the firm with the highest technology level) and a set of *followers* that share a common (lower) technology level. Firms produce differentiated varieties within the industry and compete in prices.

Within-industry demand and production. Industry i's output is a CES aggregate of the individual varieties produced by firms $j \in \mathcal{F}_i$:

$$Y_i \;=\; \left(\sum_j y_{ji}^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}, \qquad \sigma > 1,$$

where y_{ji} is the quantity of a firm j's variety and σ is the elasticity of substitution across varieties within an industry. Industry demand for Y_i comes from the final-good sector. Given the Cobb–Douglas structure, all active industries are treated symmetrically and each receives an equal expenditure share $1/N_t$ at time t.

Each firm j combines labor with its technology to produce:

$$y_{ji} = q_{ji}\, l_{ji}, \qquad q_{ji} = \lambda^{s_{ji}},$$

where l_{ji} is labor input, $s_{ji} \in \mathbb{R}_+$ is the technology level, and $\lambda > 1$ is the step size of the quality ladder. A one-step increase in s_{ji} raises firm j's productivity by a factor λ. Firms take wages and the demand system as given and set prices to maximize profits. For a given industry state (number of firms and their technology levels), price competition uniquely determines equilibrium markups and profits.

Competitive fringe in new industries. The economy features a public knowledge stock, denoted $s_t^K \in \mathbb{R}_+$, which summarizes the average level of publicly available technology. When a new industry is created at time t, it starts with a monopolist whose technology is one step ahead of the public knowledge, $s_{ji} = s_t^K + 1$, and with a competitive fringe of firms whose technology is fixed at s_t^K and who do not innovate. The fringe serves two roles. Economically, it captures the early exploratory phase of a new product line, in which many unsuccessful imitators can enter quickly using widely available technologies. Technically, the fringe ensures the monopolist faces a finite price elasticity and therefore a well-defined markup, as formalized below.

Industry states and notation. Because many industries coexist with different numbers of firms and technology gaps, it is useful to summarize the state of an industry by a small set of variables. I adopt the following notation:

- A superscript M refers to monopolies or monopolist-specific variables.
- $\hat{s}$ denotes the technology advantage of a monopolist over the fringe. (e.g. $V_{\hat{s}}^M$ is the value of a monopolist with advantage $\hat{s}$ over the fringe).
- A subscript (m, n) refers to an oligopoly in which the leader is m steps ahead of the followers and there are n followers. For example, π_{mn} denotes the leader's static profit in such an industry.
- A subscript $-mn$ refers to a representative follower in an (m, n)-industry.

Equilibrium prices, quantities, and profits within an industry are functions of this state. Given the Cobb–Douglas aggregation in the final-good sector, each active industry sells a constant amount $\bar{Y}$ of the final good in equilibrium and I take $\bar{Y}$ as the scale of industry-level revenues.

Pricing decisions. I define a firm's markup as the ratio of its price to marginal cost, $\eta \equiv p/(w/q)$, where w is the wage and q is the firm's productivity. At time t, consider a monopolist with relative advantage $\hat{s}$ over the fringe, born at some date $\tau < t$. Given industry demand $\bar{Y}$ and the presence of the competitive fringe, the monopolist's static profit-maximization problem yields:

$$\eta_{\hat{s}}^{M} = \lambda^{\hat{s}}, \tag{3}$$

$$y^{M}(\tau) = \frac{\bar{Y}}{w}\,\lambda^{s_\tau^K}, \tag{4}$$

$$\Pi_{\hat{s}}^{M} = \pi_{\hat{s}}^{M}\,\bar{Y}, \qquad \pi_{\hat{s}}^{M} = 1 - \lambda^{-\hat{s}}. \tag{5}$$

The markup is increasing in the monopolist's lead $\hat{s}$, and the profit share $\pi_{\hat{s}}^{M}$ rises from $1-\lambda^{-1}$ toward one as the monopolist pulls further ahead of the fringe. The presence of the fringe keeps demand elastic and delivers a well-defined, finite markup for any finite $\hat{s}$.

In oligopolies, firms engage in static price competition given their technology levels. Let p_{mn} denote the leader's price in an industry with gap m and n followers, and p_{-mn} the common price of a follower. Define the relative price of the leader as

$$\phi_{mn} \equiv \frac{p_{mn}}{p_{-mn}}.$$

Given the CES demand system within an industry and the Cobb–Douglas allocation of expenditure across industries, the resulting static pricing game has a unique symmetric Nash equilibrium in prices. The next lemma characterizes the equilibrium markups, quantities, and profits as functions of the state (m, n).

Lemma 3.1 (Pricing decisions in an oligopoly)**.** *Consider an oligopoly characterized by a technology gap m between the leader and the followers and n followers, all with productivity λ^{s_f}. Let $\bar{Y}$ denote normalized industry output and w the wage. In the unique symmetric Nash equilibrium in prices, the markups, outputs, and profits of the leader (denoted by subscript mn) and a representative follower (denoted by subscript $-mn$) are:*

- Markups*:*

$$\eta_{mn} = \frac{\sigma n + \phi_{mn}^{1-\sigma}}{(\sigma-1)n},$$
$$\eta_{-mn} = \frac{\sigma\big(\phi_{mn}^{1-\sigma} + n - 1\big) + 1}{(\sigma-1)\big(\phi_{mn}^{1-\sigma} + n - 1\big)}.$$

- Outputs*:*

$$y_{mn}(s_f) = \frac{(\sigma-1)n\,\phi_{mn}^{1-\sigma}\,\lambda^{m+s_f}}{\big(\sigma n + \phi_{mn}^{1-\sigma}\big)\big(\phi_{mn}^{1-\sigma} + n\big)}\,\frac{\bar{Y}}{w},$$
$$y_{-mn}(s_f) = \frac{(\sigma-1)\big(\phi_{mn}^{1-\sigma} + n - 1\big)\lambda^{s_f}}{\big(\sigma(\phi_{mn}^{1-\sigma} + n - 1) + 1\big)\big(\phi_{mn}^{1-\sigma} + n\big)}\,\frac{\bar{Y}}{w}.$$

- Profits*:*

$$\Pi_{mn} = \pi_{mn}\,\bar{Y}, \qquad \Pi_{-mn} = \pi_{-mn}\,\bar{Y}, \tag{6}$$
$$\pi_{mn} = \frac{1}{\sigma n \phi_{mn}^{\sigma-1} + 1}, \qquad \pi_{-mn} = \frac{1}{\sigma\big(\phi_{mn}^{1-\sigma} + n - 1\big) + 1}. \tag{7}$$

Moreover, the equilibrium price ratio ϕ_{mn} *is uniquely defined by*

$$\phi_{mn}^{n}\lambda^{m}\left(\sigma + \frac{1}{\phi_{mn}^{1-\sigma} + n - 1}\right) \;=\; \sigma n + \phi_{mn}^{1-\sigma} \tag{8}$$

for all $m \in \{0, 1, \dots\}$ *and* $n \in \{1, 2, \dots\}$*.*

The lemma implies several intuitive comparative statics. For any fixed number of followers n, the leader's relative price ϕ_{mn} declines as the leader moves further ahead (m increases): the more advanced the leader's technology, the lower the price it can charge relative to followers while still capturing most of the market. For any fixed technology gap m, ϕ_{mn} decreases in n: as more followers enter, competition forces the leader's relative price closer to that of its rivals. Correspondingly, the leader's markup η_{mn} rises with m and falls with n, whereas the followers' markup η_{-mn} falls in both dimensions. Leader profits π_{mn} increase when the leader pulls ahead or when there are fewer followers, while follower profits π_{-mn} shrink as the gap widens or as additional followers crowd the market.

3.3 Innovation, Industry Creation, and Destruction

Firms invest in R&D to improve their technology. Innovation is uncertain and arrives at Poisson rates that depend on firms' R&D choices. At the same time, new potential entrants arrive and decide whether to join existing industries or create new ones. Industries exit exogenously. This subsection describes these processes and the implied channels of creative destruction and knowledge diffusion.

Innovation and R&D costs. Consider a firm that chooses an innovation intensity $x \geq 0$. To achieve this rate, the firm must incur a flow R&D cost $R(x)\bar{Y}$ in units of the final good, where

$$R(x) \;=\; \frac{\alpha}{\psi}x^{\psi}, \qquad \alpha > 0,\ \psi > 1. \tag{9}$$

The parameter ψ governs the curvature of the R&D technology and the scale parameter α controls the overall costliness of innovation.

Conditional on choosing x, innovations arrive according to a Poisson process with intensity x. Each successful innovation raises the firm's technology index s_{ji} by one step, $s_{ji} \;\to\; s_{ji}+1$, and thus, $q_{ji} \;\to\; \lambda q_{ji}$. For followers in an oligopoly, I assume that knowledge is shared instantaneously among all followers: when any follower innovates, the technology level of all followers in that industry increases by one step. Thus, from the perspective of an individual follower, the effective innovation intensity includes both its own effort and the efforts of its peers. This captures the idea that some incremental improvements are easily codified and diffused among firms using similar, relatively mature technologies.

Potential entrants and industry creation. In each existing industry, a potential entrant arrives at Poisson rate $z > 0$. Upon arrival, the potential entrant chooses between:

1. *Joining an existing industry*: the entrant adopts the lowest technology level among active producers in that industry (the followers' level in an oligopoly, or the monopolist's level in a monopoly), becoming either a new follower or a neck-and-neck competitor. This reflects the idea that older technologies are easier to access and that entry into established markets typically occurs at the lower end of the quality spectrum.

2. *Creating a new industry*: the entrant starts a fresh industry as a monopolist at technology level s^K+1, one step ahead of the current public knowledge stock s^K. Creating a new industry requires paying a sunk cost $\kappa\bar{Y}$ in units of the final good, where κ is drawn from a distribution G with support $[\underline{\kappa}, \overline{\kappa}]$, $\underline{\kappa} \geq 0$, before the arrival. A lower draw κ represents a more promising or more easily implementable idea for a new product line.

The potential entrant compares the expected value of joining an existing industry with that of paying $\kappa\bar{Y}$ to found a new industry that starts with a temporary monopoly position. This entry decision jointly determines the number of firms in each industry, the rate at which new industries appear, and thus the balance between intensifying competition within incumbent markets and expanding the variety of industries in the economy.

Public knowledge and knowledge diffusion. Let s^K denote the *public knowledge stock*, defined as the average of the lowest technology levels across all industries. This object summarizes the level of widely available knowledge that can be accessed by new entrants when they design a new product. When a new industry is created, its initial technology s^K+1 depends on the current value of s^K and, conversely, as industries experience innovation and as followers catch up, s^K gradually increases.

Knowledge diffusion in the model operates through three channels:

- spillovers among followers within an industry when any follower innovates;
- adoption of incumbent technology by potential entrants that join existing industries;
- creation of new industries at $s^K + 1$ based on the evolving public knowledge stock.

These channels link firm-level innovation decisions to the evolution of the cross-sectional distribution of industries and to the growth of s^K over time.

Industry exit. Each active industry exits the economy at an exogenous Poisson rate $\delta > 0$. When an industry exits, all firms in that industry disappear and their products cease to be available in the final-good aggregator. Industry exit captures obsolescence of product lines due to shifts in tastes, technologies, or regulations that are not explicitly modeled.

Before turning to the dynamic equilibrium, it is useful to note that simple parameter restrictions nest two familiar benchmark environments. First, if I shut down industry creation and destruction by setting $z = 0$ and $\delta = 0$ while keeping $\lambda > 1$, the mass of industries is fixed and the model reduces to a multi-industry step-by-step quality-ladder framework in which growth is driven solely by within-industry innovation. Second, if I instead set $\lambda = 1$ so that quality does not improve but keep $z > 0$, innovation takes the form of pure variety creation, as in expanding-variety models. The baseline environment studied below maintains both $\lambda > 1$ and $z > 0$, so that aggregate growth reflects a combination of quality improvements within industries and the creation of new industries over time.

4 Analysis of a Stationary Equilibrium

This section characterizes a stationary and symmetric Markov-perfect equilibrium and studies its implications for industry dynamics and aggregate growth. Firm policies depend only on the current state of their industry, the cross-sectional distribution of industries over states is stationary, and key aggregates such as the public knowledge stock and the mass of industries grow at constant rates. I first describe the value functions and innovation decisions of monopolists, leaders, followers, and potential entrants. I then characterize the implied stationary distribution of industries and the aggregate growth rate.

4.1 Value Functions

The household's Euler equation,

$$\frac{\dot{C}_t}{C_t} = r_t - \rho,$$

will be used below to connect firm values to aggregate growth. On the firm side, let $V^M_{\hat{s}}$ denote the value of a monopolist with relative advantage $\hat{s}$ over the fringe, and let V_{mn} and V_{-mn} denote the values of a leader and a follower in an oligopoly with technology gap m and n followers.

Because all flow payoffs in a given state are proportional to the common revenue scale $\bar{Y}_t$, it is convenient to work with normalized values $v_t \equiv V_t/\bar{Y}_t$. In a stationary equilibrium these normalized values are time-invariant, so I write $V = v\bar{Y}$ and work directly with v throughout this section (see Appendix B.1).

Monopolist. Consider a monopolist with relative technological advantage $\hat{s}$ over the fringe. Its normalized value $v^M_{\hat{s}}$ solves

$$(\rho + g_N)\, v^M_{\hat{s}} = \max_{x\geq 0} \Big\{ \pi^M_{\hat{s}} - R(x) + x\big(v^M_{\hat{s}+1} - v^M_{\hat{s}}\big) + z\,\theta_M\big(v_{01} - v^M_{\hat{s}}\big) - \delta\, v^M_{\hat{s}} \Big\}, \tag{10}$$

where g_N is the growth rate of the mass of industries, $\pi^M_{\hat{s}}$ is the monopolist's static profit share, $R(x)$ is the R&D cost function defined above, and θ_M denotes the probability that a potential entrant who meets a monopoly chooses to join the industry rather than create a new one. The term $x(v^M_{\hat{s}+1} - v^M_{\hat{s}})$ captures the expected gain from a successful innovation that raises the monopolist one step further ahead of the fringe. The term $z\,\theta_M(v_{01} - v^M_{\hat{s}})$ captures the expected loss when a potential entrant chooses to join the industry and turns the monopoly into an industry with a leader and one follower. The last term $\delta v^M_{\hat{s}}$ reflects the risk of exogenous industry exit.

The optimal innovation intensity is

$$x_{\hat{s}}^{M} = \left(\frac{\max\{v_{\hat{s}+1}^{M} - v_{\hat{s}}^{M},\, 0\}}{\alpha} \right)^{\frac{1}{\psi-1}}.$$

Leader and follower. In an industry with a leader m steps ahead of n followers, the leader's normalized value v_{mn} satisfies

$$(\rho + g_N)v_{mn} = \max_{x\geq 0} \Big\{ \pi_{mn} - R(x) + x\big(v_{m+1,n} - v_{mn}\big) + n x_{-mn}\big(v_{m-1,n} - v_{mn}\big) + z\,\theta_{mn}\big(v_{mn+1} - v_{mn}\big) - \delta v_{mn} \Big\}, \tag{11}$$

where x_{-mn} is the common innovation intensity of followers and θ_{mn} is the probability that a potential entrant who encounters an (m, n)-industry decides to join it as a follower rather than start a new industry. The term $x(v_{m+1,n} - v_{mn})$ captures the benefit from moving further ahead of the followers, while the term $nx_{-mn}(v_{m-1,n} - v_{mn})$ reflects the loss when followers innovate and narrow the gap. Entry of an additional follower occurs at rate $z\theta_{mn}$ and lowers the leader's continuation value from v_{mn} to v_{mn+1}, and exogenous exit arrives at rate δ.

Given follower behavior, the optimal innovation intensity of the leader is

$$x_{mn} = \left(\frac{\max\{v_{m+1,n} - v_{mn},\, 0\}}{\alpha} \right)^{\frac{1}{\psi-1}}.$$

Each follower's value v_{-mn} solves

$$(\rho + g_N)v_{-mn} = \max_{x\geq 0} \Big\{ \pi_{-mn} - R(x) + \big(x + (n-1)x_{-mn}\big)\big(v_{-m+1,n} - v_{-mn}\big) + x_{mn}\big(v_{-m-1,n} - v_{-mn}\big) + z\,\theta_{mn}\big(v_{-mn+1} - v_{-mn}\big) - \delta v_{-mn} \Big\}, \tag{12}$$

and the common innovation intensity of followers is

$$x_{-mn} = \left(\frac{\max\{v_{-m+1,n} - v_{-mn},\, 0\}}{\alpha} \right)^{\frac{1}{\psi-1}}.$$

When $m = 0$ and $n \geq 2$, firms are neck-and-neck competitors. In that case, the value of a representative firm v_{0n} satisfies

$$(\rho + g_N)v_{0n} = \max_{x \geq 0} \Big\{ \pi_{0n} - R(x) + x\big(v_{1n} - v_{0n}\big) + n x_{0n}\big(v_{-1,n} - v_{0n}\big) + z\,\theta_{0n}\big(v_{0,n+1} - v_{0n}\big) - \delta v_{0n} \Big\}. \tag{13}$$

The structure of these Bellman equations is standard in step-by-step innovation models. Leaders benefit from pulling ahead and suffer when followers catch up, while followers benefit from their own and their peers' innovation and lose when the leader moves further ahead. Entry and exit shift the number of followers in the industry and truncate the horizon, and the Poisson innovation structure ensures that all continuation values are linear in the normalized scale $\bar{Y}_t$.

Potential entrant. I assume that the cost of creating a new industry, κ, is drawn from a distribution G with support $[\underline{\kappa}, \bar{\kappa}]$, where $0 \leq \underline{\kappa} < \bar{\kappa} < v_1^M$ in equilibrium. Thus even the highest-cost potential entrant would obtain strictly positive surplus from starting a new industry.

A potential entrant with cost draw κ chooses between joining an existing industry and creating a new one. If it encounters a monopoly, joining yields continuation value v_{01}, whereas creating a new industry yields $v_1^M - \kappa$. The entrant joins the monopoly if and only if $v_{01} \geq v_1^M - \kappa$. If it encounters an industry in state (m, n), joining as a follower yields v_{-mn+1}, while creating a new industry again yields $v_1^M - \kappa$. In that case, the entrant joins the incumbent industry if and only if $v_{-mn+1} \geq v_1^M - \kappa$.

These choices can be represented by indicator functions

$$\iota_\kappa^M = \begin{cases} 1 & \text{if } v_{01} \geq v_1^M - \kappa, \\ 0 & \text{otherwise,} \end{cases} \qquad \iota_{mn,\kappa} = \begin{cases} 1 & \text{if } v_{-mn+1} \geq v_1^M - \kappa, \\ 0 & \text{otherwise.} \end{cases}$$

Aggregating over κ using its distribution G yields the joining probabilities

$$\theta_M \equiv \int \iota_\kappa^M \, dG(\kappa), \qquad \theta_{mn} \equiv \int \iota_{mn,\kappa} \, dG(\kappa),$$

which are the probabilities that a potential entrant who meets a monopoly or an (m, n)-industry, respectively, decides to join rather than create a new industry.

The next lemma summarizes key properties of the value functions and innovation rates in a stationary equilibrium.

Lemma 4.1 (Properties of value functions and innovation rates)**.** *In any stationary Markov perfect equilibrium, the normalized value functions and innovation rates satisfy:*

(i) For any normalized value v,

$$0 < v < \frac{1}{\rho + \gamma}.$$

(ii) The monopolist's value $v_{\hat{s}}^M$ increases in its technology lead $\hat{s}$, with

$$\lim_{\hat{s} \to \infty} v_{\hat{s}}^M = \frac{1 + z\theta_M v_{01}}{\rho + \gamma + z\theta_M}.$$

The associated innovation rate $x_{\hat{s}}^M$ is strictly positive and decreases with $\hat{s}$, and satisfies $\lim_{\hat{s} \to \infty} x_{\hat{s}}^M = 0$.

(iii) For any $m \geq 0$, the values v_{mn}, v_{-mn} and their corresponding innovation rates x_{mn}, x_{-mn} converge to 0 as $n \to \infty$. For fixed n, we have

$$\lim_{m \to \infty} v_{mn} = \frac{1}{\rho + \gamma}, \qquad \lim_{m \to \infty} v_{-mn} = \lim_{m \to \infty} x_{mn} = \lim_{m \to \infty} x_{-mn} = 0.$$

(iv) There exists a finite upper bound $n^* \geq 1$ on the number of followers such that entry into an industry with $n \geq n^*$ followers is never profitable. In particular, $\theta_{mn} = 0$ for all m and all $n \geq n^*$.

Lemma 4.1 characterizes how market structure shapes innovation incentives. Part (i) ensures that normalized firm values are finite and bounded by the present value of a unit profit stream. Part (ii) shows that monopolists gain from increasing their lead over the fringe, but the marginal benefit of an additional step becomes smaller as the gap grows, so innovation effort eventually declines. Part (iii) shows that, for any fixed technology gap, sufficiently crowded industries deliver negligible profits and innovation in the limit, and that when the leader is far ahead, followers' values and innovation rates are negligible even though the leader's value approaches the upper bound. Part (iv) implies that this congestion effect endogenously caps industry size: beyond a finite number of followers, further entry is no longer profitable.

4.2 Stationary Markov-Perfect Equilibrium

Given the optimal decisions described in the previous subsection, I now characterize a stationary distribution of industries across states. Let $\mu_{\hat{s}}^M$ denote the share of monopoly industries whose leader is $\hat{s}$ steps ahead of the fringe, and let μ_{mn} denote the share of oligopoly industries with technology gap m and n followers. These shares satisfy

$$\sum_{\hat{s}=1}^{\infty} \mu_{\hat{s}}^{M} \;+\; \sum_{m=0}^{\infty}\sum_{n=1}^{n^*} \mu_{mn} \;=\; 1.$$

Let $\mu_M = \sum_{\hat{s}=1}^{\infty} \mu_{\hat{s}}^{M}$ denote the total share of monopolies. The rate of variety creation is then

$$\gamma \;=\; z\left[(1-\theta_M)\mu_M + \sum_{n=1}^{n^*}\sum_{m=0}^{\infty}(1-\theta_{mn})\mu_{mn}\right].$$

The Kolmogorov forward equations that determine $\mu_{\hat{s}}^{M}$ and μ_{mn} in a stationary equilibrium are given in the appendix.

Each industry disappears at rate δ, and its relative share also declines at rate g_N due to the expanding mass of industries. At the same time, γN new monopoly industries are created per unit of time, each starting at relative technology level $\hat{s} = 1$.

Monopoly industries evolve in two ways. First, they move up the ladder from $\hat{s}$ to $\hat{s}+1$ when the monopolist innovates at rate $x_{\hat{s}}^{M}$. Second, a monopoly turns into a neck-and-neck duopoly when a potential entrant joins at rate $z\theta^{M}$. Once an industry becomes an oligopoly, its gap can widen or narrow. The leader innovates at rate x_{mn}, which increases the gap by one step, while a follower innovates at total rate nx_{-mn}, which narrows the gap by one step. In addition, the number of followers can increase at rate $z\theta_{mn}$ when new entrants choose to join the industry.

The evolution of the mass of industries N is governed by

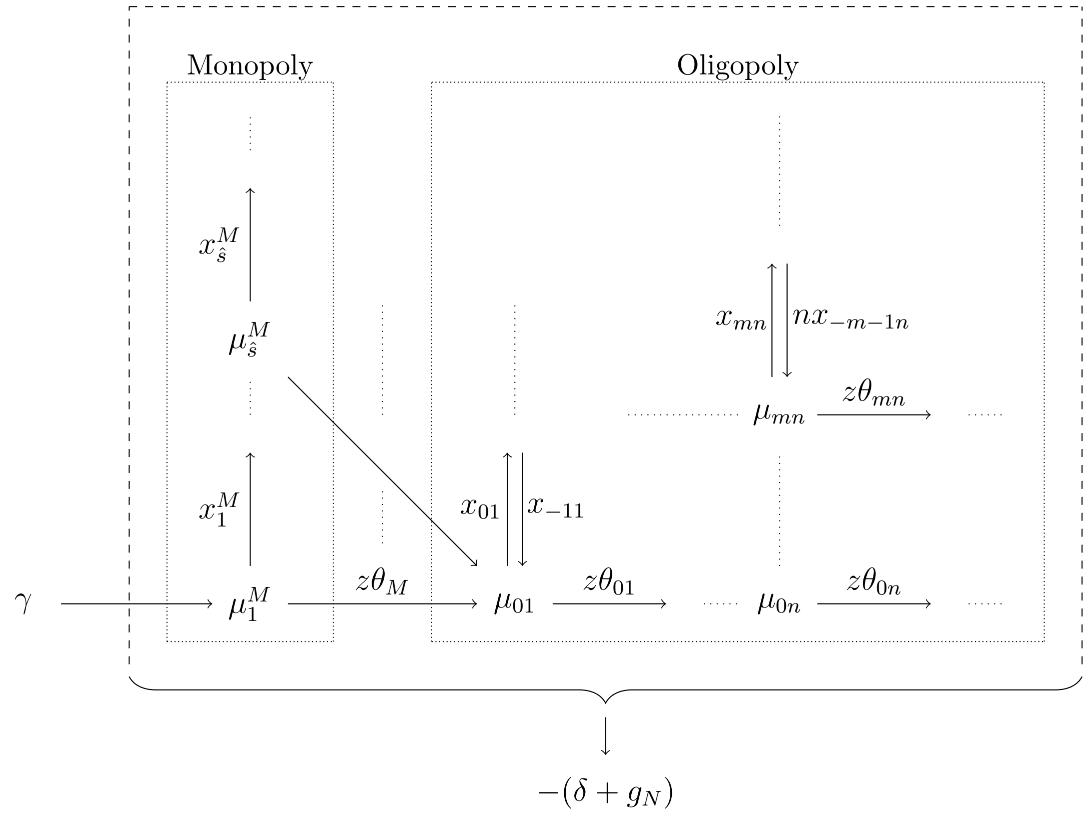
$$g_N = \gamma - \delta,$$

the net rate of variety creation. Figure 1 summarizes these flows and the resulting transitions between monopoly and oligopoly states. In a stationary equilibrium, every individual industry follows a stochastic life-cycle: it is born as a monopoly, accumulates followers and technology gaps through innovation and entry, and eventually exits. The stationary distribution $\{\mu_{\hat{s}}^{M}, \mu_{mn}\}$ describes the cross-section of industries across these life-cycle states. The key aggregate state variables for growth are therefore the rate of variety creation γ and the distribution of gaps and followers summarized by $\{\mu_{\hat{s}}^{M}, \mu_{mn}\}$.

Using the value functions, optimal policies, and the stationary distribution over industry states, I now define the equilibrium formally.

Definition 4.2 (Equilibrium)**.** *A stationary and symmetric Markov-perfect equilibrium consists of values* $\{v_{\hat{s}}^{M}, v_{mn}, v_{-mn}\}$*, policies* $\{x_{\hat{s}}^{M}, x_{mn}, x_{-mn}, \theta^{M}, \theta_{mn}\}$*, an industry distribution*

Figure 1: Flowchart of industry shares



$\{\mu^M_{\hat{s}}\}_{\hat{s}} \cup \{\mu_{mn}\}_{m,n}$, *and scalars* $\{g_N, \gamma\}$ *such that all individual decisions solve the corresponding optimization problems and the laws of motion for industry shares are satisfied.*

Let $\bar{s}^M_t$ denote the average monopolist technology level and $\bar{s}^{cf}_t$ the average technology level of the associated competitive fringe firms. For oligopoly industries, let $\bar{s}^f_t$ denote the average lowest technology level (followers and neck-and-neck competitors) and $\bar{s}^l_t$ the average highest technology level. Define the average technology gap among oligopolies as $\bar{m}$. The public knowledge stock can then be written as

$$s^K_t = \mu_M \bar{s}^M_t + (1 - \mu_M)\bar{s}^f_t.$$

The next lemma describes how these technology levels evolve in a stationary equilibrium.

Lemma 4.3 (Properties of technology levels in a stationary equilibrium)**.** *The public knowledge stock grows at a constant rate*

$$\dot{s}^K = \gamma + \sum_{\hat{s}=1}^{\infty} \mu^M_{\hat{s}} x^M_{\hat{s}} + \sum_{n=1}^{n^*} \sum_{m=1}^{\infty} n x_{-mn} \mu_{mn}.$$

The average technology levels satisfy

$$\gamma(\bar{s}_t^M - \bar{s}_t^{cf}) = \gamma + \sum_{\hat{s}=1}^{\infty} \mu_{\hat{s}}^M x_{\hat{s}}^M,$$

$$\begin{aligned}\gamma(1-\mu_M)(\bar{s}_t^l - \bar{s}_t^f) &= \gamma(1-\mu_M)\bar{m} \\ &= \sum_{n=1}^{n^*}\left[(n+1)x_{0n}\mu_{0n} + \sum_{m=1}^{\infty} x_{mn}\mu_{mn}\right] - \sum_{n=1}^{n^*}\sum_{m=1}^{\infty} n x_{-mn}\mu_{mn}.\end{aligned}$$

In a stationary equilibrium, the public knowledge stock grows at a constant rate due to new industry creation and within-industry innovation. Both monopolist and follower technology levels grow over time, while the average technology gap between leaders and followers, $\bar{m}$, remains constant. The first expression in Lemma 4.3 shows that $\dot{s}^K$ is the sum of variety creation and innovation by monopolists and followers, while the second expression decomposes the same object using the evolution of leaders' technology levels and the stationary gap $\bar{m}$. These relationships underpin the aggregate growth decomposition in the next subsection.

4.3 Aggregate Growth with Industry Dynamics

In this section, I characterize the aggregate growth rate in a stationary equilibrium that reflects different stages of the industry life-cycle. I then show several properties of the dynamics of industries in an equilibrium.

The following proposition describes the relationship between aggregate growth and industry dynamics.

Proposition 4.4 (Steady growth with non-stationary industry life-cycle)**.** *A symmetric stationary Markov perfect equilibrium exists. In the equilibrium, the constant growth rate can be expressed as:*

$$\begin{aligned} g &= \ln\lambda\left(\gamma + \sum_{\hat{s}=1}^{\infty}\mu_{\hat{s}}^M x_{\hat{s}}^M + \sum_{n=1}^{n^*}\sum_{m=1}^{\infty} n x_{-mn}\mu_{mn}\right) \\ &= \ln\lambda\left(\gamma + \sum_{\hat{s}=1}^{\infty}\mu_{\hat{s}}^M x_{\hat{s}}^M + \sum_{n=1}^{n^*}\left((n+1)x_{0n}\mu_0 + \sum_{m=1}^{\infty} x_{mn}\mu_{mn}\right) - \gamma(1-\mu_M)\bar{m}\right) \\ &= \ln\lambda\cdot\dot{s}^K. \end{aligned}$$

The first line expresses the growth rate as the step size $\ln\lambda$ times the instantaneous change in the average lowest technology level across industries. With rate γ, a new monopolist with level s^K+1 appears, but the s^K component is offset by the disappearance of incumbent industries at rate $\gamma = \delta + g_N$. The terms involving $x_{\hat{s}}^M$ and x_{-mn} capture the contributions of innovation by monopolists, neck-and-neck competitors, and followers. The second line rewrites the same object in terms of the evolution of the leaders' technology levels and the stationary gap $\bar{m}$, using Lemma 4.3. The last equality simply notes that this rate of change in the average technology level is, by definition, $\dot{s}^K$.

Proposition 4.4 shows that aggregate growth is fully determined by industry dynamics: variety creation, innovation in monopoly industries, and catch-up innovation in oligopolies. The balanced growth path arises even though individual industries experience non-stationary life-cycles, because the stationary distribution over industry states makes the average contribution of each margin constant over time.

The existence of a stationary equilibrium relies on two features of the model. First, the presence of multiple followers in each industry creates scope for catch-up by laggards and prevents leaders from drifting arbitrarily far ahead in finite time. Second, endogenous industry creation continually adds new, innovation-intensive industries to the cross-section. Together, these forces imply that, among surviving industries, the average technology gap tends to widen over time and the average innovation and entry rates decline, even though the aggregate growth rate remains constant. The next proposition formalizes these life-cycle properties.

Proposition 4.5 (Properties of industry life-cycle)**.** *In a stationary Markov perfect equilibrium, conditional on industry survival, as time progresses:*

1. *the expected growth rate of a monopolist's technology level decreases,*
2. *the expected technology gap in oligopolies increases asymptotically,*
3. *the expected innovation rate, expected entry rate, and probability of leadership turnover decrease asymptotically (Abernathy and Utterback, 1978; Klepper, 1996),*
4. *the expected price decrease asymptotically when the wage is constant, while the expected output increases when the total mass of variety is constant (Gort and Klepper, 1982; Filson, 2001).*

As technology gaps tend to widen over time, followers become less likely to catch up and leaders find it easier to maintain dominance. Both parties therefore reduce their innovation efforts, and entry into mature industries becomes rare. This leads to a gradual decline in innovation intensity, entry, and leadership turnover, and thus a slowdown in the expected growth rate of each industry. The resulting patterns—high entry and innovation in early stages, followed by consolidation and declining activity—match the main empirical regularities documented in the industry life-cycle literature. In summary, despite each industry following a non-stationary life-cycle with an eventual growth slowdown, the overlapping dynamics of many industries and the continual creation of new ones generate steady aggregate economic growth.

4.4 Constrained efficiency of industry creation

The equilibrium entry decision in Section 4.1 is formulated as a binary choice for each potential entrant with a realized cost draw κ: join the encountered incumbent industry or create a new one. It is convenient to rewrite this decision in terms of state-specific cost thresholds. Let S index an industry state, where $S = M$ denotes a monopoly with relative advantage $\hat{s}$ and $S = mn$ denotes an oligopoly with gap m and n followers. Let μ_S be the stationary share of industries in state S and let v^f_{S+} denote the follower value after joining that industry (for example v_{01} if the industry was a monopoly, or v_{-mn+1} if it was an oligopoly with gap m and n followers).

Before being randomly assigned to an incumbent industry and before observing its own cost draw κ, a potential entrant chooses a vector of thresholds $\{\hat{\kappa}^{CE}_S\}_S$ that solves

$$\max_{\{\hat{\kappa}_S\}_S} \sum_S \mu_S \left[\int_{\underline{\kappa}}^{\hat{\kappa}_S} \left(v^M_1 - \kappa \right) dG(\kappa) + \int_{\hat{\kappa}_S}^{\overline{\kappa}} v^f_{S+} \, dG(\kappa) \right], \tag{14}$$

where G is the distribution of market initiation costs and v^M_1 is the value of a new monopolist.[2] The firm commits to creating a new industry when $\kappa < \hat{\kappa}_S$ and to joining the incumbent industry otherwise. The first order condition for an interior threshold in state S is

$$\mu_S G'(\hat{\kappa}^{CE}_S) \left[-\hat{\kappa}^{CE}_S + v^M_1 - v^f_{S+} \right] = 0, \tag{15}$$

so that whenever $\mu_S G'(\hat{\kappa}^{CE}_S) > 0$ the equilibrium cutoff satisfies

$$\hat{\kappa}^{CE}_S = v^M_1 - v^f_{S+} \approx \frac{\pi^M_1 - \pi^f_{S+}}{\rho + \gamma}. \tag{16}$$

In each state, the entrant is indifferent at the cutoff between paying $\hat{\kappa}^{CE}_S$ to start a new industry and entering as a follower. The implied incumbent joining probability is

$$\theta_S \; = \; 1 - G\big(\hat{\kappa}^{CE}_S\big) \tag{17}$$

and it coincides with the probabilities defined in the entry problem above.

To assess how the equilibrium allocation of entrants across new and existing industries

[2] I assume $\underline{\kappa} < v^M_1$ so that starting a new industry yields strictly positive surplus at low costs.

compares to a socially desirable benchmark, I consider a constrained planner who can only intervene in the variety creation margin. The planner takes incumbent pricing and innovation policies as given and chooses a vector of thresholds $\{\hat{\kappa}_S^{SP}\}_S$ to maximize the representative household's lifetime utility, subject to the aggregate resource constraint and the induced law of motion for the industry distribution.

For any given set of thresholds $\{\hat{\kappa}_S\}_S$, the induced entry probabilities $\{\theta_S\}_S$ determine a unique stationary distribution over industry states and a corresponding balanced growth path for consumption. Let $\mathcal{R}(\{\hat{\kappa}_S\}_S)$ denote the ratio of incumbent R&D expenditure to aggregate output and $\mathcal{K}(\{\hat{\kappa}_S\}_S)$ the ratio of market initiation costs to output as defined in Appendix B.3, and $g(\{\hat{\kappa}_S\}_S)$ the implied aggregate growth rate. With log preferences, the planner's welfare can be written as

$$W\big(\{\hat{\kappa}_S\}_S\big) = \frac{1}{\rho}\ln\big(1 - \mathcal{R}\big(\{\hat{\kappa}_S\}_S\big) - \mathcal{K}\big(\{\hat{\kappa}_S\}_S\big)\big) + \frac{1}{\rho^2}\, g\big(\{\hat{\kappa}_S\}_S\big), \tag{18}$$

where the first term captures the level effect through net consumption and the second term captures the effect of long run growth.

The constrained planner's first order condition can be written as

$$\frac{z}{\rho}\,\mu_S G'\big(\hat{\kappa}_S^{SP}\big)\left[-\frac{1}{1-\mathcal{R}-\mathcal{K}}\,\hat{\kappa}_S^{SP} + \frac{1}{\rho}\ln\lambda\right] + \Phi_S = 0, \tag{19}$$

where all objects are evaluated at $\{\hat{\kappa}_S^{SP}\}_S$, and Φ_S collects the marginal welfare effect of changing industry composition and incumbent innovation rates. The first bracketed term is the direct effect of shifting the cutoff in state S, weighted by the mass of potential entrants throughout history, $\frac{z}{\rho}\mu_S$.

Comparing (19) with the equilibrium condition (15) shows that several forces drive a wedge between the planner's cutoff $\hat{\kappa}_S^{SP}$ and the private cutoff $\hat{\kappa}_S^{CE}$. First, focusing on the firm-level direct effect in (19), there is a *congestion externality* distorting the cost valuation. While private firms treat the nominal cost as an expense in output units, the planner applies a shadow value, $\frac{1}{1-\mathcal{R}-\mathcal{K}}$, to account for the aggregate resource constraint. Higher aggregate investment raises the marginal utility of consumption, making the utility cost of funding variety creation strictly higher than its output-unit cost.

For the second and third effects, it is useful to contrast the private marginal gain $v_1^M - v_{S+}^f \approx \frac{\pi_1^M - \pi_{S+}^f}{\rho+\gamma}$ with the planner's term $\frac{\ln\lambda}{\rho}$. Potential entrants maximize the flow of appropriable rents, $\pi_1^M - \pi_{S+}^f$, whereas the planner values the permanent expansion of the knowledge level by a quality step λ. This gives rise to a *knowledge externality*, where the social return to creating a new industry, via higher long-run growth, differs from the private rent captured by the marginal entrant. Since $\ln \lambda > \pi_1^M = 1 - \lambda^{-1}$ for $\lambda > 1$, the potential entrants inherently undervalue the contribution of the new variety to the aggregate knowledge stock.

This wedge is amplified by a *business-stealing externality.* The private discount rate, $\rho + \gamma$, is strictly larger than the social discount rate, ρ. As new industries arrive at rate γ and erode incumbent market shares, private firms discount future profits more heavily. The planner, however, discounts knowledge accumulation only at the time preference rate ρ, recognizing that the productivity gain from innovation ($\ln \lambda$) is permanent and survives the firm itself.

Finally, there is an *equilibrium adjustment effect.* The term Φ_S also embeds the general equilibrium response of values, policies, and industry shares when the entire vector of thresholds is perturbed. A local change in $\hat{\kappa}_S$ affects not only the flow of entrants into state S but also the induced evolution of other states through the stationary distribution and the aggregate resource constraint. This adjustment further shifts the planner's preferred cutoff away from the private one. In the next section, I evaluate these conditions in the calibrated economy and quantify the resulting entry wedge.

5 Quantitative Analysis

This section uses computational methods to study the quantitative implications of the model. I first calibrate a stationary equilibrium of the economy to match key moments for the U.S. economy over the 1982–2011 period. I then use the calibrated model to examine how changes in the parameters governing entry and competition shape industry dynamics and aggregate growth. Finally, I evaluate the constrained efficiency benchmark from Section 4.4 in the calibrated economy and compare two innovation policies that subsidize incumbent R&D and variety creation.

For the quantitative analysis, I parameterize the distribution of market initiation costs as follows. Let κ denote the cost of creating a new industry, and write

$$\kappa = k\, \tilde{v}_1^M,$$

where k is drawn from a transformed beta distribution with support $[0, 0.9]$ and $\tilde{v}_1^M$ is the value of a monopolist in the baseline equilibrium.[3] In comparative statics, I vary the mean of this distribution while holding its standard deviation fixed at 0.1. In the next section, I

[3]This normalization ties the support of market initiation costs to the value of a new monopolist, which is convenient for comparing cutoffs across equilibria.

Table 1: Calibrated Parameters

Externally calibrated		
ρ	0.0228	Rate of time preference (10-year Treasury rate − TFP growth)
δ	0.1500	Exit rate (brand exit rate (Broda and Weinstein, 2010))
Internally calibrated		
σ	3.8961	Elasticity of substitution
λ	1.0351	Innovation step size
α	0.0897	Multiplier of R&D cost function
ψ	1.2917	Curvature of R&D cost function
$\hat{k}$	0.0872	Mean of market initiation cost divided by baseline v_1^M
z	0.3115	Potential entrant arrival rate

evaluate these conditions in the calibrated economy and quantify the resulting entry wedge.

5.1 Calibration

I calibrate eight parameters to reflect the U.S. economy over the 30-year period starting in 1982. Table 1 reports the parameter values. Two parameters are chosen externally. The rate of time preference ρ is set to 0.0228, the difference between the average 10-year Treasury rate and average TFP growth, where TFP is measured using the utilization-adjusted series from Fernald (2014). The industry exit rate δ is set to 0.15, based on the median 1-year brand exit rate in Broda and Weinstein (2010).

The remaining six parameters—$\{\sigma, \lambda, \alpha, \psi, \hat{k}, z\}$—are estimated jointly (details are provided in the appendix), targeting the empirical moments in Table 2. While all parameters simultaneously affect all moments in general equilibrium, I provide a heuristic argument for which data moments primarily identify each parameter.

The elasticity of substitution σ governs the intensity of static price competition, while the market initiation cost parameter $\hat{k}$ governs the extensive margin of entry. Both capture the level and dispersion of within-industry competition, and I identify σ and $\hat{k}$ by matching the average and standard deviation of cost-weighted markups. I compute these markup moments using Compustat data and the production approach from De Loecker, Eeckhout and Unger (2020) with input weights, which is more comparable to the product-level environment of the model.

To identify the R&D cost parameters (α, ψ), which govern the cost of innovation for incumbents, I target the average R&D-to-GDP ratio and the average innovation frequency. The R&D-to-GDP ratio comes from NCSES (National Center for Science and Engineering Statistics , NCSES), and the innovation frequency from Cai and Tian (2024), covering 1981–

Table 2: Targeted moments

	Data	Model	Source
Average TFP growth	0.0091	0.0091	Fernald (2014)
Average cost-weighted markup	1.3667	1.3717	Compustat, De Loecker, Eeckhout and Unger (2020)
S.D. of cost-weighted markup	0.2904	0.2872	Compustat, De Loecker, Eeckhout and Unger (2020)
Average R&D/GDP	0.0255	0.0252	NCSES (NSF), 2025
Average innovation frequency	0.5195	0.5198	Cai and Tian (2024) (1981–2014)
1-year median brand entry rate	0.1618	0.1586	Broda and Weinstein (2010) (1994, 1999–2003)

2014. In the model, both incumbent R&D spending and market creation costs are counted as R&D, since both represent forms of innovation.

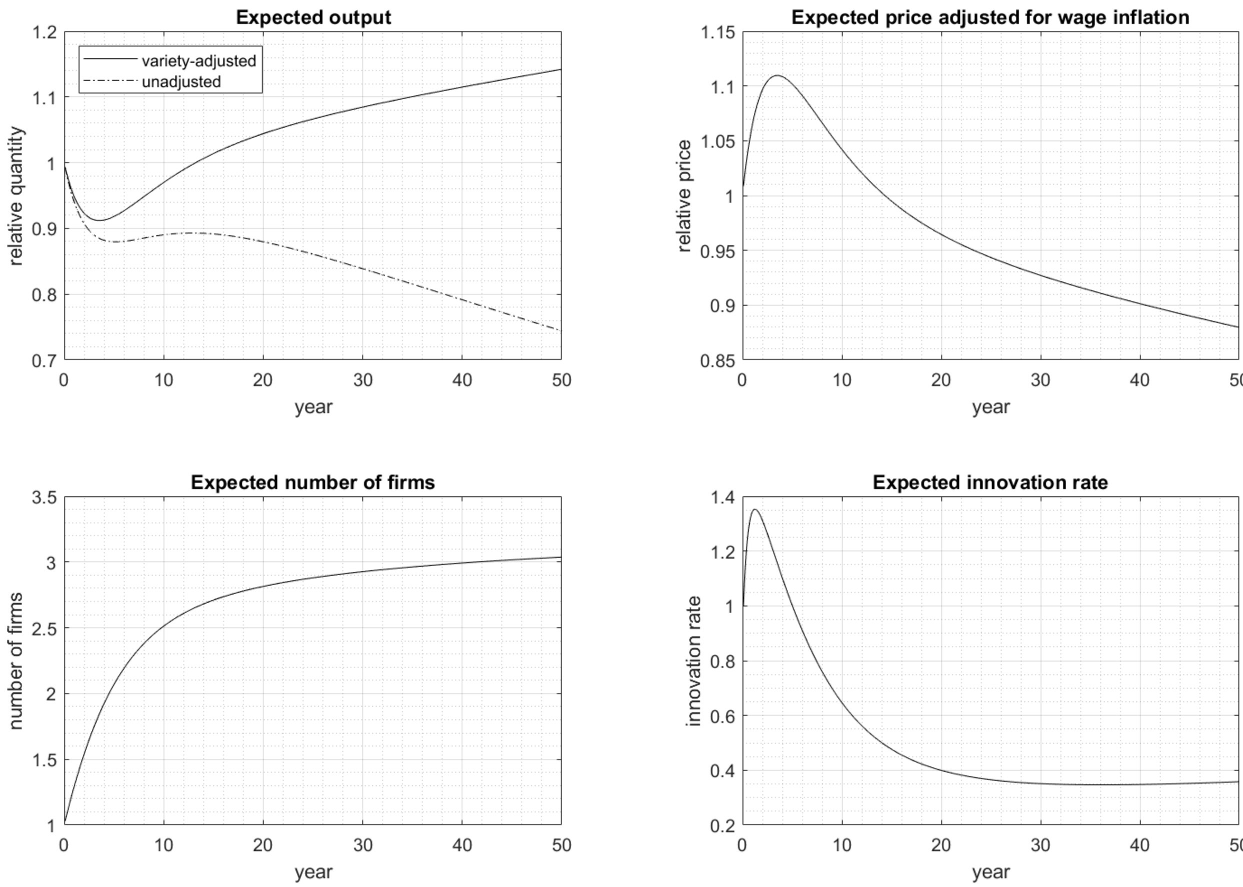
The innovation step size λ determines the return to innovation and is the primary driver of the aggregate growth rate. The parameter is calibrated to match the average TFP growth rate. Finally, z governs the frequency of potential entrant shocks and is identified by the median brand entry rate.

5.2 Industry Life-Cycle Patterns

In the model, industries follow non-stationary life-cycles: they start small and highly innovative, then grow in size while innovation activity gradually slows. The profiles in Figure 2 show that the calibrated economy exhibits this pattern in a way that is qualitatively consistent with the industry life-cycle evidence discussed in Section 2. Each panel shows the expected change in an industry variable over a 50-year period, conditional on survival. Details of the calculation are provided in the appendix.

The top-left panel plots changes in output relative to the initial level. I report two measures. *Unadjusted output* (dash-dotted line) corresponds to the industry-level aggregator in the model and is best interpreted as the output of a narrowly defined market, close to a group of brands that compete directly. *Variety-adjusted output* (solid line) adjusts for changes in effective market size due to between-industry business-stealing effects from variety creation. In practice, new varieties often do not immediately generate new industry classifications, so business stealing is more salient at the brand level than at the industry level. The unadjusted series therefore aligns more closely with brand-level dynamics, while the adjusted series is more comparable to industry-level outcomes. As documented by Gort and Klepper (1982), Jovanovic and MacDonald (1994), and Filson (2001), industry-level output tends to rise over time, whereas brand-level output tends to decline as newer products capture market share—a pattern emphasized by Argente, Lee and Moreira (2024). The model reproduces both trends: variety-adjusted output rises over the life-cycle, while unadjusted

Figure 2: Industry Life-Cycle Patterns



output eventually declines as new varieties compete away demand.

The top-right panel shows the expected change in industry prices, adjusted for wage inflation, relative to the initial level. I define the industry price of industry i as

$$P_i = \left(\sum_{j \in i} p_{ji}^{1-\sigma} \right)^{1/(1-\sigma)} .$$

Dividing by the wage accounts for aggregate inflation. Consistent with the evidence, industry prices fall over the life-cycle as innovation reduces unit costs and competition intensifies.

The bottom panels plot the expected number of firms and the expected innovation rate over time. As industries evolve, firm counts increase while innovation rates decline (Abernathy and Utterback, 1978; Filson, 2001; Jovanovic and Wang, 2024). This trajectory captures the canonical pattern of early expansion with frequent entry, followed by consolidation and lower innovation in more mature industries. The model does not feature an explicit shakeout phase with declining firm counts as in Klepper (1996), but such a pattern could be generated by introducing a fixed operating cost: as technology gaps widen and follower profits fall, some firms would optimally exit.

These age profiles are not directly targeted in the calibration in Table 2, so their qualitative similarity to the empirical life-cycle patterns is a useful check on the mechanism.

5.3 Constrained Efficiency of Calibrated Economy

I now use the constrained planner framework of Section 4.4 to assess the efficiency of variety creation in the calibrated economy. The goal is to quantify the wedge between the decentralized entry cutoffs $\{\hat{\kappa}_S^{CE}\}_S$ and the planner's cutoffs $\{\hat{\kappa}_S^{SP}\}_S$ that maximize welfare subject to the equilibrium laws of motion.

The planner's first-order condition for the cutoff in state S, equation (19), combines three components: (i) a congestion effect through the shadow value of aggregate consumption $1/(1-\mathcal{R}-\mathcal{K})$, (ii) a knowledge effect through the term $(\ln\lambda)/\rho$, and (iii) a composition term Φ_S that captures how reallocating entrants across states affects the cross-sectional distribution of industries and incumbent innovation incentives. In this subsection, I evaluate these objects at the competitive equilibrium.

First, by construction the planner attaches a higher utility cost to market initiation than private entrants: the shadow value of aggregate consumption is scaled by $1/(1-\mathcal{R}-\mathcal{K}) > 1$, so any given resource cost is magnified relative to the private valuation. In the calibrated economy, this congestion effect is modest but non-trivial: for a given cutoff $\hat{\kappa}$, the planner's effective valuation of the market initiation cost is 2.59% higher than the private valuation.

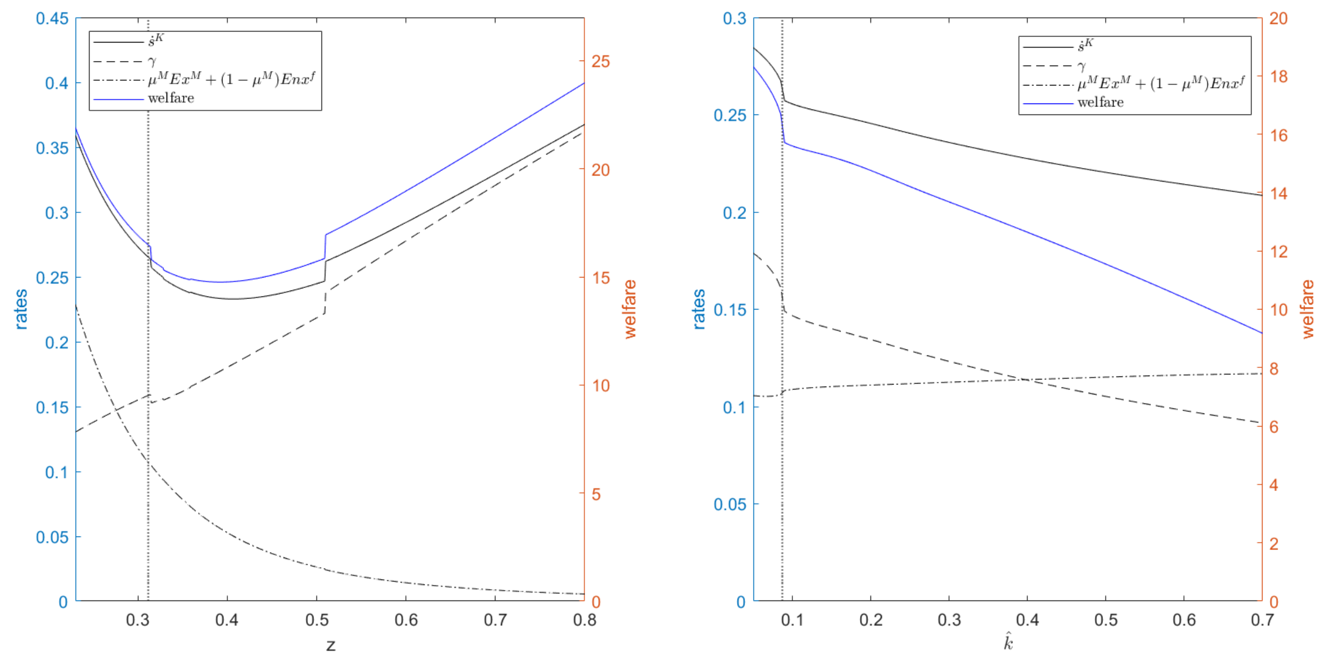
Second, the knowledge component pushes in the opposite direction and dominates in the baseline equilibrium. Although the private value v_1^M exceeds the simple bound $\pi_1^M/(\rho+\gamma)$ because it includes the option value of future innovation, the net private incentive $v_1^M - v_{S+}^f$ remains strictly below the social benefit $(\ln\lambda)/\rho$ for any state with positive mass ($\mu_S > 0$). Ignoring higher-order composition effects, the planner's cutoff would satisfy

$$\hat{\kappa}^{SP} = \frac{1-\mathcal{R}-\mathcal{K}}{\rho}\ln\lambda.$$

Under the current calibration, this expression evaluates to 1.47, which is strictly above the maximum cost $\overline{\kappa} = 0.57$ in the support of G. Conditional on holding innovation rates and industry shares fixed, it is therefore socially desirable to bear higher market initiation costs and create more varieties than in the competitive equilibrium.

Third, I quantify the full equilibrium wedge by computing the derivative of welfare with respect to the state-contingent thresholds at the competitive equilibrium. Evaluating the

Figure 3: Comparative Statics: Changing z and $\hat{k}$



planner's first-order condition at the decentralized cutoffs and numerically approximating the derivatives, I find that they are positive for all economically relevant states ($\mu_S > 2\%$): for any state with substantial mass, variety creation is chosen less frequently than the constrained optimum. When I recompute these derivatives while keeping innovation rates and industry shares fixed at their competitive values—so that only the direct externalities enter and the composition terms Φ_S are suppressed—they remain positive for all active states, consistent with the comparison above between $v_1^M - v_{S+}^f$ and $(\ln \lambda)/\rho$. The equilibrium adjustment effect therefore does not overturn the sign of the wedge. In the calibrated economy, variety creation is too low relative to the constrained planner's benchmark. Potential entrants are too willing to enter existing oligopolies to chase short-term business-stealing rents, and too reluctant to pay the fixed costs to open new markets.

5.4 Comparative Statics

To understand how industry dynamics shape productivity growth, I conduct comparative statics with respect to parameters governing industry creation and competition. I first vary the potential entrant arrival rate z and the market initiation cost parameter $\hat{k}$, which control the intensity and cost of entry. I then vary the elasticity of substitution σ, which governs the strength of within-industry competition.

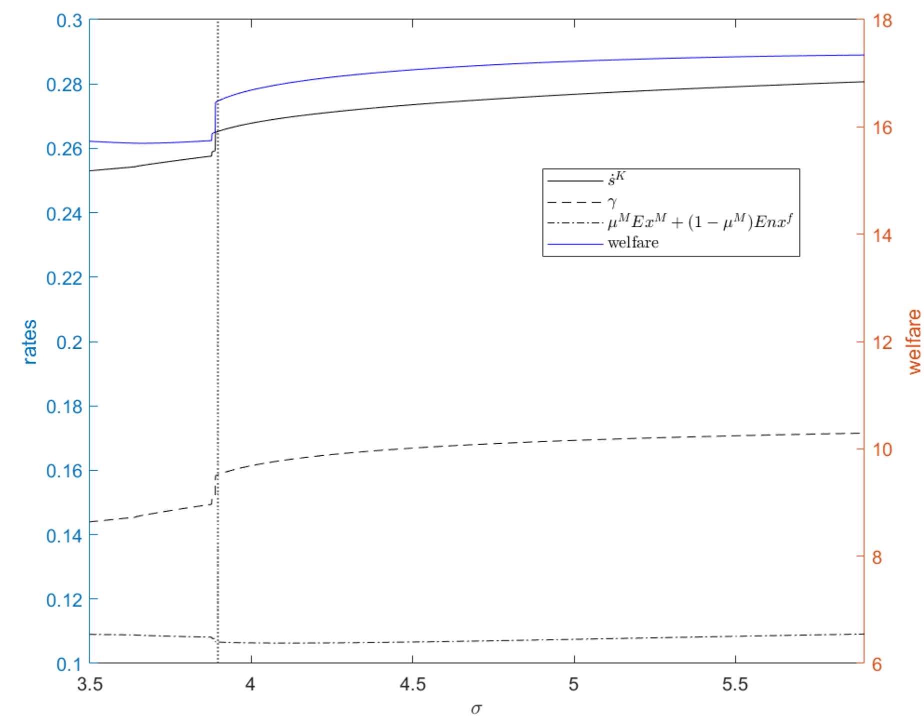
The left panel of Figure 3 shows how aggregate growth and its components respond to changes in z. As z increases, the variety creation rate γ rises, but the average innovation rates of monopolists and oligopoly followers decline. More potential entrants generate more new industries, but they also reduce incumbent profits through both between- and within-industry business-stealing effects. This lowers the expected value of innovation and crowds out R&D investment by incumbent firms. The net result is a U-shaped relationship between z and aggregate growth: when z is low, increasing it reduces growth because the crowding-out effect dominates, while further increases in z beyond a threshold raise growth as the extensive-margin benefit from greater variety creation outweighs the loss from reduced incumbent innovation. The discrete jumps visible in these series are discussed below.

The right panel of Figure 3 varies the mean market initiation cost $\hat{k}$. Reducing $\hat{k}$ uniformly lowers the cost of starting new industries and monotonically increases the growth rate. As $\hat{k}$ falls, the variety creation rate γ rises steadily. Incumbents' R&D incentives decline, but the magnitude is smaller than under changes in z, because the within-industry business-stealing channel is weaker. Leaders are affected more than followers, as the value of market leadership falls when the economy is filled with new and differentiated product lines. However, the overall reduction in R&D is muted by a shrinking average technology gap and a smaller oligopoly share. Thus, despite lower incumbent innovation and total R&D spending, growth rises due to more frequent variety creation and the entry of younger, more competitive firms.

These experiments also have a simple policy interpretation. The U-shaped relationship between z and growth cautions against the view that "more entrepreneurs" are always the right response to low growth. In the model, indiscriminately increasing the mass of potential entrants can move the economy into a region where additional entry mainly intensifies business stealing in existing industries and crowds out high-value incumbent R&D, with only limited gains in variety creation. What matters for long-run growth is not just the quantity of startups but their composition: growth is highest when entry takes the form of new-industry creation rather than incremental entry into incumbent markets. Reducing the effective initiation cost $\hat{k}$ is a more targeted way to tilt entry toward new industries and raise growth, but it is also harder to implement in practice, since $\hat{k}$ reflects deep features of the business environment—regulation, fixed setup costs, and institutional barriers—that are difficult to change quickly.

Figure 4 reports comparative statics for the elasticity of substitution σ. As σ increases, competition intensifies within industries and the leader position becomes more valuable. The net effect on incumbent innovation is modest and depends on the industry distribution. In industries with small technology gaps, stronger competition encourages both leaders and followers to invest more aggressively. In industries with large gaps, higher σ discourages

Figure 4: Comparative Statics: Changing σ



follower innovation because catching up becomes less attractive. In the calibrated economy, the average incumbent innovation rate decreases slightly when σ is low and increases slightly when σ is high, but the magnitude of these changes is small.

In contrast, the effect on variety creation is more pronounced. As it becomes less attractive to enter existing industries as a follower, potential entrants are more likely to initiate new industries instead. The variety creation rate γ rises with σ, and this extensive-margin effect dominates the small changes in incumbent innovation, so the aggregate growth rate increases. This pattern underscores a complementarity between pro-competitive product-market policies and technological diversification in the model.

From a theoretical perspective, the muted and sign-ambiguous response of incumbent innovation to σ reflects the importance of industry composition: the effect of tougher competition depends on the distribution of industries over technology gaps and life-cycle stages. This echoes the findings of Aghion et al. (2005), who show that the competition-innovation relationship depends on firms' distance to the technological frontier. In my environment, however, the main action comes from the extensive margin: stronger competition within industries pushes potential entrants away from mature sectors and toward the creation of new industries, raising γ and, through this channel, the aggregate growth rate. In this sense, the analysis adds a complementary mechanism to the competition-and-innovation literature, highlighting that competition can foster growth not only by stimulating "escape competition" innovation by incumbents, but also by accelerating structural change through variety creation.

5.4.1 Discontinuities in Comparative Statics

The comparative statics in Figures 3 and 4 exhibit several discrete jumps in growth and innovation outcomes as parameters vary. These discontinuities arise from the discrete industry state space and the bounded support of market initiation costs, not from numerical instability.

Entry decisions are governed by threshold comparisons among value functions: for a given distribution of costs, a potential entrant compares the value of joining an incumbent industry in state S with the value of creating a new industry. As parameters change, the ordering of values such as v_{02} and v_1^M can switch. Because G has bounded support, the economy can hit a corner in which the entry condition for a given state moves from interior to corner, shifting the probability of entry in that state from strictly positive to zero (or vice versa) for the entire cost distribution. These nonlinear switches in state-contingent entry generate abrupt changes in the variety creation rate γ. Since γ enters the effective discount rate $(\rho + g_N)$, these shifts feed back into firm values, generating the discrete changes in aggregate rates and welfare visible in the figures. This feature reflects the inherently discrete and non-linear nature of industrial evolution in the model.

5.5 Policy Experiments: Subsidizing Innovation

I now compare two policy interventions that subsidize innovation activity in the economy. Both policies are financed through lump-sum taxes on the representative household:

1. *Policy 1 (P1): Rewarding successful R&D investment.* A firm receives a subsidy equal to $b_r \Delta V$ upon a successful innovation, where $b_r \in [0, 1)$ is the subsidy rate and ΔV is the value increase resulting from the innovation.

2. *Policy 2 (P2): Rewarding variety creation.* A potential entrant receives a subsidy equal to $b_i V_1^M$ if it chooses to start a new industry, where $b_i \in [0, 1)$ is the subsidy rate and V_1^M is the value of a new monopolist.

Policy 1 directly targets incumbent R&D, aiming to boost technological improvement within existing industries. Policy 2 instead targets the extensive margin by increasing the returns to successful variety creation.

Figure 5: Policy Comparison: Rewarding Incumbent R&D and Variety Creation

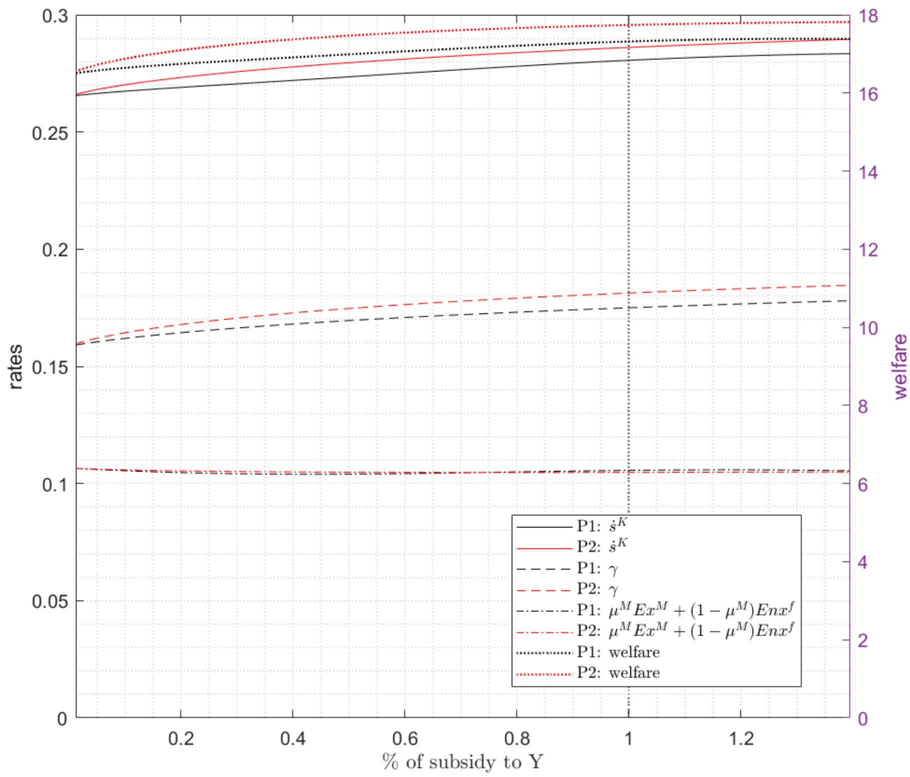


Figure 5 shows comparative statics for both policies across a range of subsidy-to-output ratios. Neither policy substantially changes the average innovation rate of incumbent firms. The marginal returns to R&D are already low in mature, follower-heavy industries, and the subsidies only partially offset the business-stealing effects of entry. Both policies, however, increase the rate of variety creation, with Policy 2 generating a much stronger response. This difference arises because P2 directly raises the payoff from starting a new industry, whereas P1 affects variety creation only indirectly through its effect on firm values.

To compare the two policies at a common fiscal cost, I choose subsidy rates such that government expenditures on each policy amount to 1% of output in the stationary equilibrium. Under Policy 1, this corresponds to rewarding $b_r \approx 0.611$ of the value increase from a successful incumbent innovation. At this level, the long-run growth rate rises by about 5.8 basis points and welfare increases by 5.0%. Under Policy 2, the 1% of output constraint implies a much lower subsidy rate, $b_i \approx 0.087$ of the value of a new monopolist. Even with this smaller subsidy, Policy 2 delivers a larger impact: the growth rate increases by about 7.7 basis points and welfare rises by 7.6%.

For any given level of fiscal expenditure, the policy that rewards variety creation is therefore more effective at raising both growth and welfare. It fosters a higher rate of technological renewal by encouraging entry into new markets, sustaining aggregate growth through ongoing industry turnover. In contrast, subsidizing incumbent R&D yields smaller gains, in part because the benefits are diluted by business stealing and declining innovation incentives in mature industries. These results highlight a key implication of the model: directing innovation policy toward the extensive margin of industry creation can be more growth-enhancing than traditional R&D subsidies focused on incumbents, even when the latter are calibrated to be quite generous on a per-innovation basis.

6 Conclusion

This paper has developed a Schumpeterian growth framework in which industries follow endogenous life-cycles and aggregate growth emerges from the overlapping dynamics of these industries. Industries are born as monopolies, become oligopolies with intense competition and leadership turnover, and eventually mature as technology gaps widen and innovation incentives decline. Despite this non-stationary behavior at the industry level, the economy admits a stationary Markov-perfect equilibrium in which the cross-sectional distribution of industries is constant and aggregate productivity grows at a constant rate.

A central contribution is to derive an explicit growth decomposition that links aggregate TFP growth to industry dynamics. The long-run growth rate can be written as the sum of three components: variety creation through the birth of new industries, frontier innovation by monopolists, and catch-up innovation by followers within oligopolies. Each term is expressed in terms of equilibrium innovation intensities and the stationary distribution of industries over life-cycle states. This provides a Harberger-style map from the cross-section of industries to aggregate growth and clarifies how stable long-run growth can coexist with front-loaded innovation and declining business dynamism at the industry level.

The quantitative analysis shows that the parameters governing entry and competition shape both industry dynamics and aggregate growth in systematic ways. Varying the potential entrant arrival rate z yields a U-shaped relationship between startup entry and growth: when z is low, additional entry mainly intensifies business stealing in existing industries and crowds out high-value incumbent R&D, whereas beyond a threshold the extensive-margin benefit from variety creation dominates. Lowering the market-initiation cost $\hat{k}$ tilts entry more directly toward new industries and monotonically raises growth, although changing $\hat{k}$ in practice would require shifting deep features of the business environment. Increases in the elasticity of substitution σ have modest, composition-dependent effects on incumbent innovation but a clear effect on the extensive margin: tougher within-industry competition pushes entrants away from mature sectors and toward starting new industries, raising the rate of variety creation and, through this channel, the aggregate growth rate.

The constrained-efficiency analysis and policy experiments highlight the role of entry wedges and the target of innovation policy. Potential entrants undervalue variety creation because they focus on appropriable rents, discount future profits at a rate that includes the creative-destruction hazard, and ignore the aggregate resource constraint. Consistent with this, for a given fiscal cost, subsidies that reward the creation of new industries generate larger gains in long-run growth and welfare than equally costly subsidies to incumbent R&D: policies aimed at the extensive margin "enable" structural renewal by seeding new, innovation-intensive industries, whereas incumbent R&D subsidies are diluted by business stealing and low marginal returns in mature industries.

More broadly, the analysis suggests that understanding the sources of long-run growth requires taking seriously the life-cycle of industries and the extensive margin of industry creation, not just the intensive margin of R&D within existing markets. The framework is deliberately parsimonious, but it provides a tractable laboratory for studying changes in business dynamism, the interaction between competition and innovation, and the design of policies that shape the structure and pace of economic growth.

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A Proofs

A.1 Lemma 3.1

From the maximization problem, the demand for firm j in industry i is $y^d_{ji} = \frac{p_{ji}^{-\sigma}}{\sum_j p_{ji}^{1-\sigma}}\bar{Y}$. The expressions for markups, outputs, and profits are straightforward from the firm's problem given demand and the price competition. Most of the properties follow from algebra.

1. Following equation can be derived by rearranging the expression.

$$\phi n \lambda^m \left(\sigma + \frac{1}{\phi^{1-\sigma} + n - 1}\right) = \sigma n + \phi^{1-\sigma}.$$

   Consider ϕ as a variable for LHS and RHS while other parameters and variables being constant. As LHS is increasing while RHS is decreasing in ϕ, $\lim_{\phi\to 0}$ LHS $= 0$ while $\lim_{\phi\to 0}$ RHS $= \infty$, and $\lim_{\phi\to\infty}$ LHS $= \infty$ while $\lim_{\phi\to\infty}$ RHS $= \sigma n < \infty$, the solution uniquely exists.

3. Markup changes when m increases and follower markup decreasing in n follow from the previous property. As $\phi_{mn}\lambda^m = \eta_{mn}/\eta_{-mn}$, η_{mn} is also decreasing in n. It means that $\frac{1}{n}\phi_{mn}^{1-\sigma}$ is decreasing in n.

A.2 Lemma 4.1

1. For any normalized value v, $0 < v < \frac{1}{\rho+\gamma}$.
   A normalized profit π a firm can experience satisfies $0 < \pi < 1$ to Lemma 3.1. Therefore, for any states,

$$v < \int_0^\infty e^{-(\rho+\gamma)t} \cdot 1 dt = \frac{1}{\rho+\gamma}.$$

   If a firm never invest in R&D, the value is bigger than 0, similarly. Therefore, the sequences are bounded below by 0.

2. $v^M_{\hat{s}}$ is increasing in $\hat{s}$ with $\lim_{\hat{s}\to\infty} v^M_{\hat{s}} = \frac{1+z\theta_M v_{01}}{\rho+\gamma+z\theta_M}$ and $x^M_{\hat{s}}$ is positive and decreasing in $\hat{s}$ with $\lim_{s\to\infty} x^M_{\hat{s}} = 0$.
   For a given s, assume $v^M_{\hat{s}+1} \leq v^M_{\hat{s}}$. Then, $x^M_{\hat{s}} = 0$, so

$$\begin{aligned}(\rho+\gamma+z\theta_M)v^M_{\hat{s}} &= \pi^M_{\hat{s}} + z\theta_M v_{01} \\ &< \pi^M_{\hat{s}+1} - R(x^M_{\hat{s}+1}) + x^M_{\hat{s}+1}(v^M_{\hat{s}+2} - v^M_{\hat{s}+1}) + z\theta_M v_{01} \\ &= (\rho+\gamma+z\theta_M)v^M_{\hat{s}+1}.\end{aligned}$$

Sum of the second and third terms in the RHS of the inequality is weakly positive, as the sum can be zero when $x^M_{\hat{s}+1} = 0$. This is an contradiction, so $v^M_{\hat{s}+1} > v^M_{\hat{s}}$ for any s. $\{v^M_{\hat{s}}\}_{s=1}^{\infty}$ is an increasing and bounded sequence, so a limit exists. From equation (10), $\lim_{s\to} v^M_{\hat{s}} = \frac{1+z\theta_M v_{01}}{\rho+\gamma+z\theta_M}$.

The positivity of $\{x^M_{\hat{s}}\}_{s=1}^{\infty}$ comes from increasing $\{v^M_{\hat{s}}\}_{s=1}^{\infty}$. The normalized value also has a finite limit, so $\lim_{s\to\infty} x^M_{\hat{s}} = 0$.

Define $\Delta\pi^M_{\hat{s}} = \pi^M_{\hat{s}+1} - \pi^M_{\hat{s}}$, $\Delta v^M_{\hat{s}} = v^M_{\hat{s}+1} - v^M_{\hat{s}}$, and $\Lambda(\Delta v^M_{\hat{s}}) = -R(x^M_{\hat{s}}) + x^M_{\hat{s}} \Delta v^M_{\hat{s}}$. Assume $\Delta v^M_{\hat{s}} > \Delta v^M_{\hat{s}+1}$, but $\Delta v^M_{\hat{s}-1} \leq \Delta v^M_{\hat{s}}$. Then,

$$
\begin{aligned}
(\rho + \gamma + z\theta_M)\Delta v^M_{\hat{s}} &= \Delta\pi^M_{\hat{s}} + \Lambda(\Delta v^M_{\hat{s}+1}) - \Lambda(\Delta v^M_{\hat{s}}) \\
&< \Delta\pi^M_{\hat{s}-1} + \Lambda(\Delta v^M_{\hat{s}}) - \Lambda(\Delta v^M_{\hat{s}-1}) = (\rho + \gamma + z\theta_M)\Delta v^M_{\hat{s}-1}.
\end{aligned}
$$

The inequality comes from the monotonicity of Λ. Contradiction. Therefore, $\Delta v^M_{\hat{s}-1} > \Delta v^M_{\hat{s}}$. As $\{x^M_{\hat{s}}\}_{s=1}^{\infty}$ is positive and goes to 0, for any $\hat{s}$, there exists $\hat{s} > s$ such that $x^M_{\hat{s}} > x^M_{\hat{s}+1}$, hence $\Delta v^M_{\hat{s}} > \Delta v^M_{\hat{s}+1}$, so $x^M_{\hat{s}}$ decreases in $\hat{s}$.

3. For any $m \in \{0, 1, \cdots\}$, v_{mn}, v_{-mn}, x_{mn}, and x_{-mn} converge to 0 as $n \to \infty$. For any $n \in \{1, 2, \cdots\}$, v_{mn} converges to $\frac{1}{\rho+\gamma}$ while v_{-mn}, x_{mn}, and x_{-mn} converge to 0 as $m \to \infty$.
   A leader's profit always goes to 1, while a follower's profit always goes to 0 as m goes to infinity, regardless of n. As n goes to infinity, the profits go to 0 for any m. Thus the result follows.

4. A finite maximum number of followers n^* exists.
   As $v_{0n} \to 0$ when $n \to \infty$, there exists $\tilde{n}$ such that $v_{0n} \leq v^M_1 - \overline{\kappa}$ for any $n \geq \tilde{n}$. Consider a follower in state mn gets payoff $\pi_{0n'}$, where n' is the current number of followers, until it first becomes a neck-and-neck competitor, while the Markov rates of the process are the same. Let the value from such process be v^0_{-mn}. Then, if $n \geq \tilde{n}$

$$
v_{-mn} \leq v^0_{-mn} \leq \max\left\{\frac{\pi_{0n}}{\rho + \gamma}, v^M_1 - \overline{\kappa}\right\},
$$

   because any possible follower payoffs after reaching mn is smaller than π_{0n} and for any n, $v_{0n} \leq v^M_1 - \overline{\kappa}$. As π_{0n} also goes to 0 as n increases, there exists some $\hat{n}$ such that $\frac{\pi_{0\hat{n}}}{\rho+\gamma} \leq v^M_1 - \overline{\kappa}$, therefore, $v_{-m\hat{n}} \leq v^M_1 - \overline{\kappa}$ for any m. In other words, there always exists n^* such that, if $n < n^*$, there is at least one m such that $\theta_{mn} > 0$, while $\theta_{mn^*} = 0$ for any m.

A.3 Lemma 4.3

The average levels follow the following differential equations in a stationary equilibrium.

$$\mu_M \dot{\bar{s}}_t^M = -(z\theta_M + \gamma)\mu_M \bar{s}_t^M + \gamma(s_t^K + 1) + \sum_{\hat{s}}^{\infty} \mu_{\hat{s}=1}^M x_{\hat{s}}^M,$$

$$\mu_M \dot{\bar{s}}_t^{cf} = -(z\theta_M + \gamma)\mu_M \bar{s}_t^{cf} + \gamma s_t^K,$$

$$(1-\mu_M)\dot{\bar{s}}^f = -\gamma(1-\mu_M)\bar{s}^f + z\theta_M \mu_M \bar{s}^M + \sum_{n=1}^{n^*}\sum_{m=1}^{\infty} n x_{-mn}\mu_{mn},$$

$$(1-\mu_M)\dot{\bar{s}}^l = -\gamma(1-\mu_M)\bar{s}^l + z\theta_M \mu_M \bar{s}^M + \sum_{n=1}^{n^*}\left((n+1)x_{0n}\mu_0 + \sum_{m=1}^{\infty} x_{mn}\mu_{mn}\right).$$

As $\bar{s}_t^M - \bar{s}_t^{cf} = \sum_{\hat{s}=1}^{\infty} \mu_{\hat{s}}^M \hat{s}$ and $\bar{s}_t^l - \bar{s}_t^f = \bar{m} = \sum_{n=1}^{n^*}\sum_{m=0}^{\infty} \mu_{mn} m$ are constant in a stationary equilibrium, one can get the three equations in the lemma using the differential equations, the fact that $\mu_M = \frac{\gamma}{z\theta_M + \gamma}$, and $s^K = \mu_M \bar{s}^M + (1-\mu_M)\bar{s}^f$.

A.4 Proposition 4.4

Let S denote the (countable) set of industry states with unbounded technology gaps $m \in \{0, 1, 2, \dots\}$ and $1 \leq n \leq n^*$. Fix a candidate variety–creation rate $\gamma \in [0, z]$. The firms' HJBs define a discounted continuous–time Markov decision problem on S which, by standard uniformization, can be written as a discrete–time Bellman equation on $\ell_\infty(S)$ with discount factor $\beta(\gamma) \in (0, 1)$ and bounded payoffs. The associated Bellman operator is a contraction, so for each γ there exists a unique bounded value function $v(\gamma)$ and associated optimal R&D and entry policies $\theta(\gamma)$. These policies induce a generator $Q(\gamma)$ for the industry–level Markov chain on S with killing at rate $\delta > 0$. Under standard regularity conditions on $Q(\gamma)$ (bounded jump rates and irreducibility), this chain admits a unique stationary cross–sectional distribution $\mu(\gamma)$ over alive states. Define the implied variety–creation rate

$$\Gamma(\gamma) \equiv z \sum_{s \in S} \big(1 - \theta_s(\gamma)\big)\mu_s(\gamma).$$

$\Gamma : [0, z] \to [0, z]$ is continuous. By the intermediate value theorem, there exists at least one fixed point $\gamma^* \in [0, z]$ such that $\Gamma(\gamma^*) = \gamma^*$.

A monopoly industry with relative level $\hat{s}$ and was born at τ produces $Y^M(\hat{s}, \tau) = y^M(\tau) = \frac{\bar{Y}}{w}\lambda^{s_\tau^K}$ until it stops to be a monopolist. Output produced by an oligopoly industry with a technology gap of m and n followers with technology level s^f can be written as $Y_{mn}(s^f) = \tilde{Y}_{mn}^f \lambda^{s^f} \frac{\bar{Y}}{w} = \tilde{Y}_{mn}^l \lambda^{m+s^f} \frac{\bar{Y}}{w}$, where $\tilde{Y}_{mn}^f$ and $\tilde{Y}_{mn}^l$ only depend on m and n. From the labor market conditions, the growth rate of $\frac{\bar{Y}}{w} = \frac{Y}{wN}$ in a stationary equilibrium is $-g_N$.

Over time interval $[t, t + \Delta t]$, changes in industry log outputs are as follows.

$$\Delta \ln Y^M(\hat{s}, \tau) = \begin{cases} \hat{s} \ln \lambda + \ln \tilde{Y}^f_{01} = \hat{s} \ln \lambda + \ln \tilde{Y}^l_{01} & \text{with probability } z\theta_M \Delta t + o(\Delta t), \\ -\ln y^M(\tau) & \text{with probability } \delta \Delta t + o(\Delta t), \\ -g_N \Delta t & \text{otherwise,} \end{cases}$$

$$\Delta \ln Y_{0n}(s^f) = \begin{cases} \ln \tilde{Y}^f_{1n} - \ln \tilde{Y}^f_{0n} = \ln \lambda + \ln \tilde{Y}^l_{1n} - \ln \tilde{Y}^l_{0n} & \text{with probability } (n+1)x_{0n} \Delta t + o(\Delta t), \\ \ln \tilde{Y}^f_{0n+1} - \ln \tilde{Y}^f_{0n} = \ln \tilde{Y}^l_{0n+1} - \ln \tilde{Y}^l_{0n} & \text{with probability } z\theta_{0n} \Delta t + o(\Delta t), \\ -\ln Y_{0n}(s^f) & \text{with probability } \delta \Delta t + o(\Delta t), \\ -g_N \Delta t & \text{otherwise,} \end{cases}$$

$$\Delta \ln Y_{mn}(s^f) = \begin{cases} \ln \tilde{Y}^f_{m+1n} - \ln \tilde{Y}^f_{mn} = \ln \lambda + \ln \tilde{Y}^l_{m+1n} - \ln \tilde{Y}^l_{mn} & \text{with probability } x_{mn} \Delta t + o(\Delta t), \\ \ln \lambda + \ln \tilde{Y}^f_{m-1n} - \ln \tilde{Y}^f_{mn} = \ln \tilde{Y}^l_{m-1n} - \ln \tilde{Y}^l_{mn} & \text{with probability } nx_{-mn} \Delta t + o(\Delta t), \\ \ln \tilde{Y}^f_{mn+1} - \ln \tilde{Y}^f_{mn} = \ln \tilde{Y}^l_{mn+1} - \ln \tilde{Y}^l_{mn} & \text{with probability } z\theta_{mn} \Delta t + o(\Delta t), \\ -\ln Y_{mn}(s^f) & \text{with probability } \delta \Delta t + o(\Delta t), \\ -g_N \Delta t & \text{otherwise.} \end{cases}$$

In cases except for destruction of industries, outputs also decrease by $g_N \Delta t$ in addition to the changes from state transition, but it is ignored as it would not appear in the limit. Also, at each moment t, a new industry with output $Y^M(1, t)$ appears with rate γ per each industry. Therefore,

$$\frac{d}{dt} \int_0^{N_t} \ln Y_{it} di = \gamma N_t \ln Y^M(1, t) + \mu_M N_t \mathbb{E}_t \frac{d}{dt} \ln Y^M + (1 - \mu_M) N_t \mathbb{E}_t \frac{d}{dt} \ln Y_{mn}.$$

Let F^f_{mnt} be a distribution of followers' technology levels for the industries with gap m and n followers at time t and F^M_{cft} be a distribution of fringe firms' technology levels for the monopolies at t. Using the stationary distribution of technology gaps, the aggregate growth rate is as follows.

$$
\begin{aligned}
g_t &= g_N + \frac{d}{dt}\ln \bar{Y}_t = g_N - \frac{\dot{N}_t}{N_t^2}\int_0^{N_t} \ln Y_{it} di + \frac{1}{N_t}\frac{d}{dt}\int_0^{N_t} \ln Y_{it} di \\
&= g_N - g_N \ln \bar{Y}_t + \gamma \ln Y^M(1,t) - \delta \ln \bar{Y}_t - g_N + \sum_{\hat{s}=1}^{\infty} \mu_{\hat{s}}^M z\theta_M(\hat{s}\ln\lambda + \ln \tilde{Y}_{01}^f) \\
&\quad + \ln\lambda \sum_{n=1}^{n^*}\sum_{m=1}^{\infty} nx_{-mn}\mu_{mn} + \sum_{n=1}^{n^*}\Bigg(\mu_{0n}\Big[(n+1)x_{0n}(\ln \tilde{Y}_{1n}^f - \ln \tilde{Y}_{0n}^f) + z\theta_{0n}(\ln \tilde{Y}_{0n+1}^f - \ln \tilde{Y}_{0n}^f)\Big] \\
&\quad + \sum_{m=1}^{\infty}\mu_{mn}\Big[x_{mn}(\ln \tilde{Y}_{m+1n}^f - \ln \tilde{Y}_{mn}^f) + nx_{-mn}(\ln\lambda + \ln \tilde{Y}_{m-1n}^f - \ln \tilde{Y}_{mn}^f) + z\theta_{mn}(\ln \tilde{Y}_{mn+1}^f - \ln \tilde{Y}_{mn}^f)\Big]\Bigg) \\
&= \; - g_N \ln \bar{Y}_t + \gamma(\ln\frac{\bar{Y}_t}{w_t} + s_t^K \ln\lambda) + (g_N - \gamma)\ln \bar{Y}_t + z\theta_M \ln\lambda \sum_{\hat{s}=1}^{\infty}\mu_{\hat{s}}^M \hat{s} + \ln\lambda \sum_{n=1}^{n^*}\sum_{m=1}^{\infty} nx_{-mn}\mu_{mn} \\
&\quad + \ln \tilde{Y}_{01}^f(x_{-11}\mu_{11} - 2x_{01}\mu_{01} + z\theta_M\mu_M - z\theta_{01}\mu_{01}) + \cdots \\
&\quad + \ln \tilde{Y}_{mn}^f(x_{m-1n}\mu_{m-1n} + nx_{-m-1n}\mu_{m+1n} - (x_{mn} + nx_{-mn})\mu_{mn} + z\theta_{mn-1}\mu_{mn-1} - z\theta_{mn}\mu_{mn}) \\
&\quad + \cdots
\end{aligned}
$$

$$
\begin{aligned}
&= \gamma s_t^K \ln\lambda - \gamma\mu_M \ln\lambda \int_0^{\infty} s dF_{cft}^M(s) - \gamma \sum_{n=1}^{n^*}\sum_{m=0}^{\infty}\mu_{mn}\int_1^{\infty}\left(\ln \tilde{Y}_{mn}^f + s\ln\lambda\right) dF_{mnt}^f(s) \\
&\quad + z\theta_M\mu_M \bar{s}^M \ln\lambda - z\theta_M\mu_M \ln\lambda \int_0^{\infty} s dF_{cft}^M(s) + \ln\lambda \sum_{n=1}^{n^*}\sum_{m=1}^{\infty} nx_{-mn}\mu_{mn} + \gamma\sum_{n=1}^{n^*}\sum_{m=0}^{\infty}\mu_{mn}\ln \tilde{Y}_{mn}^f \\
&= \; \ln\lambda\left[\gamma s_t^K + z\theta_M\mu_M\left(\bar{s}_t^M - \bar{s}_t^{cf}\right) - \gamma\left(\mu_M \bar{s}_t^{cf} + (1-\mu_M)\bar{s}_t^f\right) + \sum_{n=1}^{n^*}\sum_{m=1}^{\infty} nx_{-mn}\mu_{mn}\right]
\end{aligned}
$$

Using $s_t^K = \mu_M \bar{s}_t^M + (1-\mu_M)\bar{s}_t^f$ and $\mu_M = \frac{\gamma}{\gamma + z\theta_M}$ in a stationary equilibrium, it can be shown that the sum of the first three terms in the bracket is equal to $\gamma\left(\bar{s}_t^M - \bar{s}_t^{cf}\right)$. From Lemma 4.3,

$$
g = \ln\lambda\left(\gamma + \sum_{\hat{s}=1}^{\infty}\mu_{\hat{s}}^M x_{\hat{s}}^M + \sum_{n=1}^{n^*}\sum_{m=1}^{\infty} nx_{-mn}\mu_{mn}\right) = \ln\lambda \cdot \dot{s}^K.
$$

Similarly,

$$
g = \ln\lambda\left(\gamma + \sum_{\hat{s}=1}^{\infty}\mu_{\hat{s}}^M x_{\hat{s}}^M + \sum_{n=1}^{n^*}\left((n+1)x_{0n}\mu_0 + \sum_{m=1}^{\infty} x_{mn}\mu_{mn}\right) - \gamma(1-\mu_M)\bar{m}\right).
$$

A.5 Proposition 4.5

1. The expected growth rate of monopolist technology level decreases.
   This is straightforward from Lemma 4.1.

2. The expected technology gap of the oligopoly industry asymptotically increases.
   Let $\mathbf{x} = (2x_{01}, \cdots, (n+1)x_{0n}, \cdots, (n^*+1)x_{0n^*}, x_{11} - x_{-11}, \cdots, x_{mn} - nx_{-mn}, \cdots)'$. From Proposition 4.4, $\mu^{O\prime}\mathbf{x} > 0$, where $\mu^O = \frac{1}{1-\mu_M} \cdot (\mu_{01}, \cdots, \mu_{0n^*}, \mu_{11}, \cdots, \mu_{mn}, \cdots)'$. Define a transition matrix of a industry conditional on survival:

$$R^O \equiv \begin{pmatrix} -2x_{01} - z\theta_{01} & z\theta_{01} & & \cdots & & 2x_{01} & \\ 0 & & \ddots & & & & \ddots \\ \vdots & & & -(n^*+1)x_{0n^*} & & & \\ x_{-11} & & & & \ddots & & \\ & & \ddots & & & & \end{pmatrix},$$

   where the transition rate from state mn to state $m'n'$ is at $(m \cdot n^* + n)$th row and $(m' \cdot n^* + n')$th column. Another transition matrix of industry destruction and creation can be defined as $\Gamma^O \equiv \gamma(E_1 - I)$, where I is an identity matrix, E_1 is a matrix with all the elements in the first column being one and the other elements zero, and I and E_1 have the same size and corresponding definitions for each element as R^O. By definition, μ^O is the stationary distribution of a Markov process generated by a transition rate matrix $R^O + \Gamma^O$. Let $\mathbf{n} = (0, 0, \cdots, 1, 1, \cdots, 2, 2, \cdots)'$ where each number repeats n^* times. Then, $\mathbf{x} = R^O\mathbf{n}$.

   Now, I can define the expected technology gap conditional on its survival after time t from its transition from a monopoly to a duopoly. $\mathbb{E}m_t = b_1' e^{R^O t}\mathbf{n}$, where b_1's first element is one and the others are zero. If $\frac{d}{dt}\mathbb{E}m_t > 0$, then the expected gap is increasing.

$$\begin{aligned} \frac{d}{dt}\mathbb{E}m_t &= b_1' e^{R^O t} R^O \mathbf{n} = b_1' e^{R^O t}\mathbf{x} \\ &= (b_1' - \mu^{O\prime} e^{\Gamma^O t}) e^{R^O t}\mathbf{x} + \mu^{O\prime} e^{\Gamma^O t} e^{R^O t}\mathbf{x} \\ &= (b_1' - \mu^{O\prime} e^{\Gamma^O t}) e^{R^O t}\mathbf{x} + \mu^{O\prime}\mathbf{x}. \end{aligned}$$

   The expression in the parentheses goes to zero as t increases because b_1 is the limit distribution of a Markov process generated by the transition rate matrix Γ^O. $e^{R^O t}\mathbf{x}$ is bounded as x_{mn} and nx_{-mn} are bounded when $n \leq n^*$. Therefore, $\lim_{t\to\infty} \frac{d}{dt}\mathbb{E}m_t = \mu^{O\prime}\mathbf{x} > 0$. The expected gap is asymptotically increasing.

3. The expected innovation rate and R&D investment asymptotically decrease.
   This is from the decreasing innovation rates of monopolists, asymptotic increase of expected gap, and the asymptotic decrease of oligopoly innovation rates.

4. The expected entry to the industry asymptotically decreases.
   This is also straightforward from the fact that the number of followers always weakly increases and that there is a absorbing state of $n = n^*$ where $\theta_{mn^*} = 0$ for any m.

B Equilibrium Definition

B.1 Hamilton-Jacobi-Bellman Equations and Normalization

The Hamilton-Jacobi-Bellman equations are as follows.

$$
\begin{aligned}
rV_{\hat{s}}^M - \dot{V}_{\hat{s}}^M &= \max_{x\in[0,\infty)} \Pi_{\hat{s}}^M - R(x)\bar{Y} + x(V_{\hat{s}+1}^M - V_{\hat{s}}^M) + z\theta_M(V_{01} - V_{\hat{s}}^M) - \delta V_{\hat{s}}^M \quad \text{for } s \in \{1, 2, \cdots\},\\
rV_{mn} - \dot{V}_{mn} &= \max_{x\in[0,\infty)} \Pi_{mn} - R(x)\bar{Y} + x(V_{m+1n} - V_{mn}) + nx_{-mn}(V_{m-1n} - V_{mn})\\
&\qquad + z\theta_{mn}(V_{mn+1} - V_{mn}) - \delta V_{mn} \quad \text{for } m, n \in \{1, 2, \cdots\},\\
rV_{-mn} - \dot{V}_{-mn} &= \max_{x\in[0,\infty)} \Pi_{-mn} - R(x)\bar{Y} + (x + (n-1)x_{-mn})(V_{-m+1n} - V_{-mn}) + x_{mn}(V_{-m-1n} - V_{-mn})\\
&\qquad + z\theta_{mn}(V_{-mn+1} - V_{-mn}) - \delta V_{-mn} \quad \text{for } m, n \in \{1, 2, \cdots\},\\
rV_{0n} - \dot{V}_{0n} &= \max_{x\in[0,\infty)} \Pi_{0n} - R(x)\bar{Y} + x(V_{1n} - V_{0n}) + nx_{0n}(V_{-1n} - V_{0n})\\
&\qquad + z\theta_{0n}(V_{0n+1} - V_{0n}) - \delta V_{0n} \quad \text{for } n \in \{1, 2, \cdots\},
\end{aligned}
$$

where θs are the weighted entry decision as previously defined.

Define $v := \frac{V}{\bar{Y}}$, with corresponding state sub/superscripts. As $\dot{V} = \dot{v}\bar{Y} + v\dot{\bar{Y}}$ and $\frac{\dot{\bar{Y}}}{\bar{Y}} = g - g_N = r - \rho - g_N$,

$$rV - \dot{V} = rv\bar{Y} - \dot{v}\bar{Y} - v\dot{\bar{Y}} = (\rho + g_N)v\bar{Y} - \dot{v}\bar{Y}.$$

In a stationary equilibrium, the RHS of normalized HJBs become time-invariant, so $\dot{v} = 0$ for any states.

B.2 Kolmogorov forward equations

The Kolmogorov forward equations for industry shares are as follows. In a stationary equilibrium, $\dot{\mu} = 0$.

$$(\delta + g_N)\mu_1^M + \dot{\mu}_1^M = -x_1^M \mu_1^M + \gamma - z\theta_M \mu_1^M, \tag{20}$$

$$(\delta + g_N)\mu_{\hat{s}}^M + \dot{\mu}_{\hat{s}}^M = x_{\hat{s}-1}^M \mu_{\hat{s}-1}^M - x_{\hat{s}}^M \mu_{\hat{s}}^M - z\theta_M \mu_{\hat{s}}^M \quad \text{for } s \geq 2, \tag{21}$$

$$(\delta + g_N)\mu_{01} + \dot{\mu}_{01} = x_{-11}\mu_{11} - 2x_{01}\mu_{01} + z\theta_M \mu_M - z\theta_{01}\mu_{01}, \tag{22}$$

$$(\delta + g_N)\mu_{0n} + \dot{\mu_{0n}} = nx_{-1n}\mu_{1n} - (n+1)x_{0n}\mu_{0n} + z\theta_{0n-1}\mu_{0n-1} - z\theta_{0n}\mu_{0n} \quad \text{for } n \geq 2, \tag{23}$$

$$(\delta + g_N)\mu_{11} + \dot{\mu_{11}} = 2x_{01}\mu_{01} + x_{-21}\mu_{21} - (x_{11} + x_{-11})\mu_{11} - z\theta_{11}\mu_{11}, \tag{24}$$

$$\begin{aligned}(\delta + g_N)\mu_{1n} + \dot{\mu_{1n}} = {} & (n+1)x_{0n}\mu_{0n} + nx_{-2n}\mu_{2n} - (x_{1n} + nx_{-1n})\mu_{1n} \\ & + z\theta_{1n-1}\mu_{1n-1} - z\theta_{1n}\mu_{1n} \quad \text{for } n \geq 2,\end{aligned} \tag{25}$$

$$\begin{aligned}(\delta + g_N)\mu_{mn} + \dot{\mu_{mn}} = {} & x_{m-1n}\mu_{m-1n} + nx_{-m-1n}\mu_{m+1n} - (x_{mn} + nx_{-mn})\mu_{mn} \\ & + z\theta_{mn-1}\mu_{mn-1} - z\theta_{mn}\mu_{mn} \quad \text{for } m \geq 1 \text{ and } n \geq 2,\end{aligned} \tag{26}$$

B.3 Market clearing conditions

$$\begin{aligned}
Y &= C + Y\left(\mathcal{R} + \mathcal{K}\right), \\
&\quad \text{where } \mathcal{R} = \sum_{\hat{s}=1}^{\infty} \mu_{\hat{s}}^M R(x_{\hat{s}}^M) + \sum_{m=0}^{\infty}\sum_{n=1}^{\infty} \mu_{mn}\left(R(x_{mn}) + nR(x_{-mn})\right) \\
&\quad \text{and } \mathcal{K} = z\left(\mu_M \int_{\underline{\kappa}}^{\hat{\kappa}_M} \kappa dG(\kappa) + \sum_{m=0}^{\infty}\sum_{n=1}^{\infty} \mu_{mn} \int_{\underline{\kappa}}^{\hat{\kappa}_{mn}} \kappa dG(\kappa)\right), \\
1 &= \frac{Y}{w}\Bigg(\sum_{\hat{s}=1}^{\infty} \mu_{\hat{s}}^M \lambda^{-\hat{s}} + \sum_{m=0}^{\infty}\sum_{n=1}^{\infty} \mu_{mn}\bigg(\frac{n(\sigma-1)\phi_{mn}^{1-\sigma}}{(\sigma n + \phi_{mn}^{1-\sigma})(\phi_{mn}^{1-\sigma} + n)} \\
&\qquad + \frac{n(\sigma-1)(\phi_{mn}^{1-\sigma} + n - 1)}{(\sigma(\phi_{mn}^{1-\sigma} + n - 1) + 1)(\phi_{m}^{1-\sigma} + n)}\bigg)\Bigg). \\
A &= Y\left(\sum_{\hat{s}=1}^{\infty} \mu_{\hat{s}}^M v_{\hat{s}}^M + \sum_{m=0}^{\infty}\sum_{n=1}^{\infty} \mu_M (v_m + v_{-m})\right).
\end{aligned}$$

C Quantitative Appendix

C.1 Computation Procedure

For computation, I need to set a maximum level a monopolist can reach and a maximum gap and a maximum number of followers that an oligopoly can achieve. The numbers I choose are

30, 50, and 30 in order which are large enough for calculating the outcomes. Main algorithm of the quantitative model is as follows.

1. Guess γ and v_{01}.

2. Solve for monopoly values.

3. Solve for oligopoly values using the new v_1^M.

4. Solve for stationary distribution and new γ.

5. Check if the new γ and v_{01} match the initial guess. If not, go back to the first step.

C.2 Transition Probability and Industry Variables

From the process of industry states, I can define a transition probability between different states after t amount of time, conditional on the industry's survival, as follows.

$$P_{x_1x_2}(t) = P(state(t+h) = x_2 | state(h) = x_1 \text{ and not destroyed until } t+h)$$

where $state(t) \in \{1, 2, \cdots, s, \cdots\} \cup \{01, 02, \cdots, mn, \cdots\}$ is the state of the firm at time t. The corresponding Kolmogorov backward and forward equations are as follows.

$$\dot{P}(t) = RP(t)$$
$$\dot{P}(t) = P(t)R$$

where R is a transition rate matrix of the process conditional of the survival of the industry. With the Kolmogorov equations and R, the transition probabilities conditional on survival can be derived as $P(t) = e^{Rt}$.